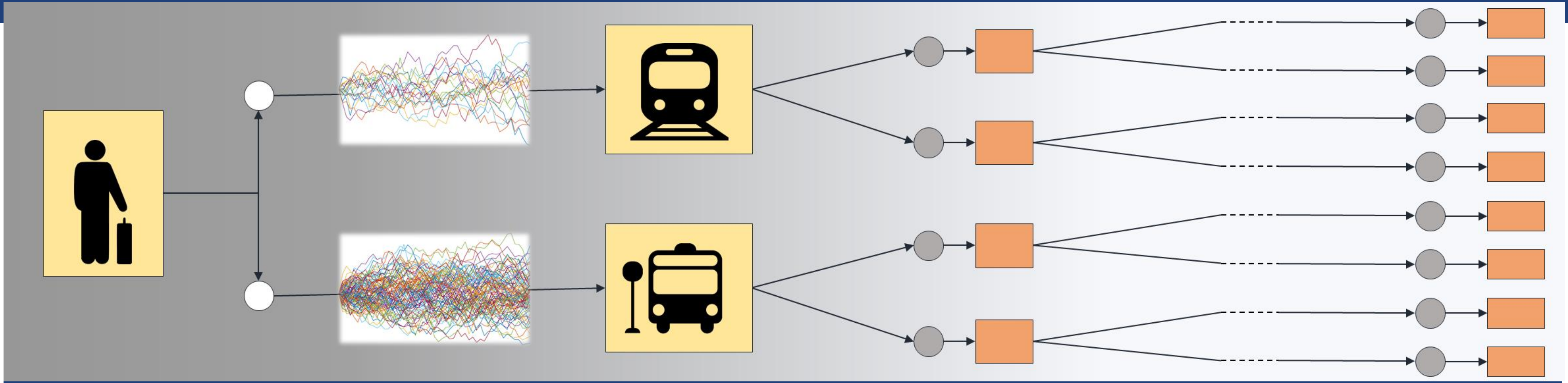

Stochastic process models of railway traffic flow:
Models, methods, and implications

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Joint work with: Francesco Corman and Mehdi Keyvan-Ekbatani

Introduction

Much effort has been devoted to model **car traffic**: How to describe traffic characteristics based on individual drivers' behavior (e.g., car-following and lane-changing)

Little exchange of ideas filtered to/from the similar problem for track-based transportation

We want to develop novel **railway traffic flow models** based on driver behavior modeling by:

- extending key ideas from car traffic
- considering the specific/different aspects of railway (e.g., the strict safety system)

Our main goals are:

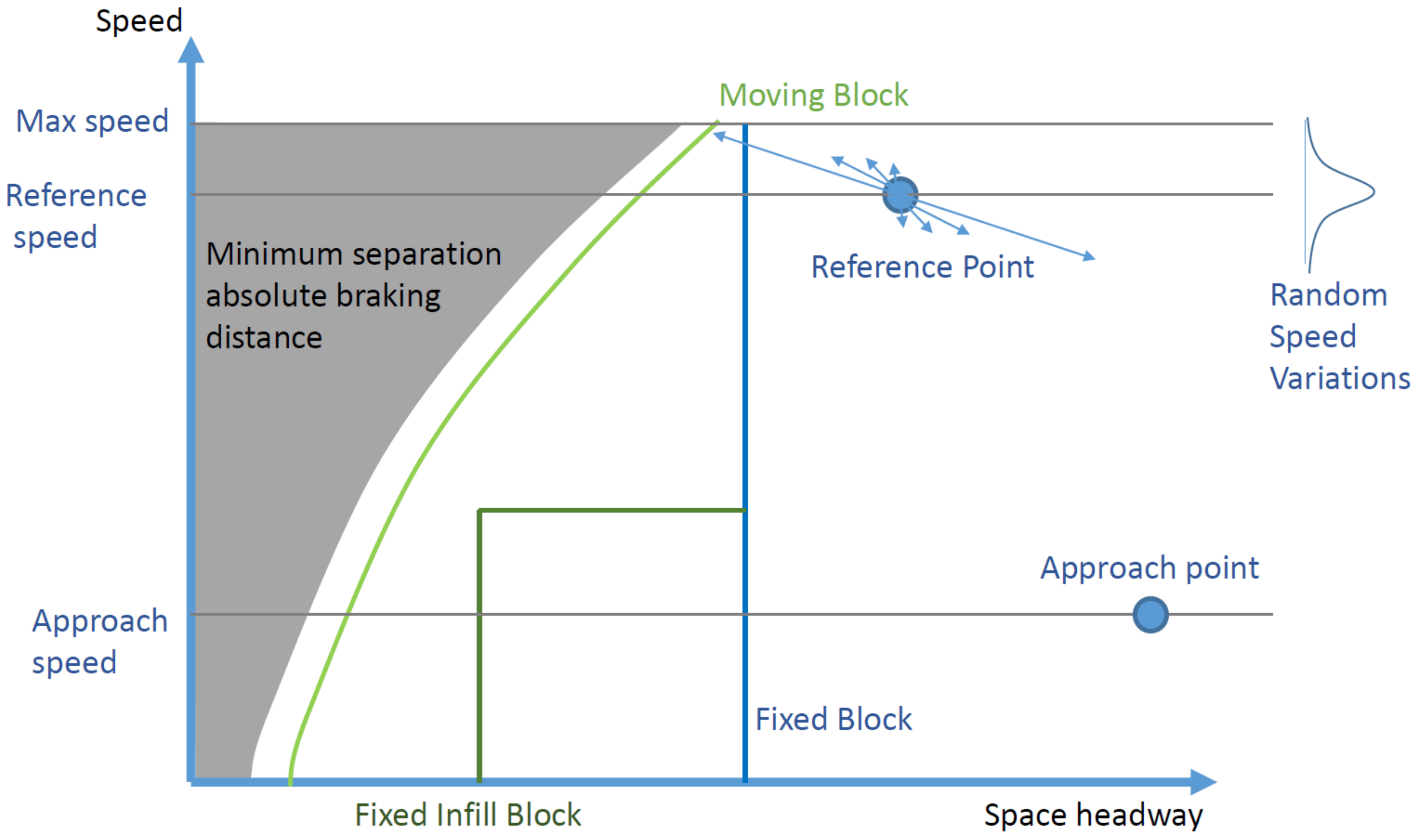
1. Formalize the relation between train driver characteristics, including his/her behavior, vehicle's technology, signaling system, and the **aggregate performance** of the system
2. Quantify the system benefit resulting from **automatic train operation (ATO)** in terms of added regularity and reliability, compared to a **human driver**

Problem description

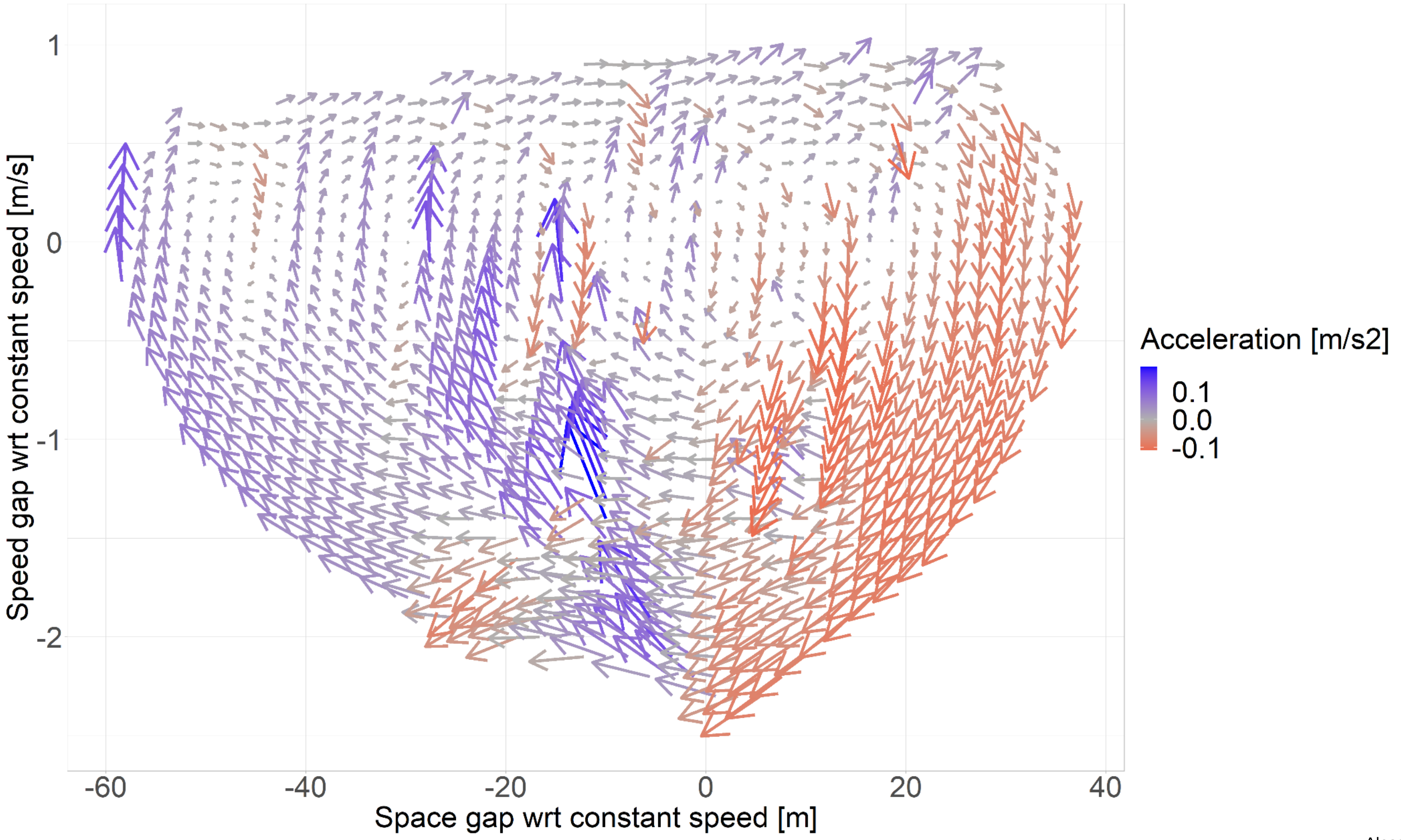
Leader-follower model

Follower is subject to speed variations

Yellow signals force the follower to decelerate



Analysis on recorded data from the Swiss network (50 trains)



Stochastic process models (1/2): BM and OU

We define 4 stochastic processes of increasing complexity that model different situations

1. Speed follows a Brownian motion **(BM)**

$$\textbf{[BM]:} \quad \begin{cases} \mathrm{d}v(t) = \sigma \mathrm{d}W(t) \\ \mathrm{d}s(t) = v(t)\mathrm{d}t \end{cases}$$

$\mathrm{d}v(t) = \sigma \mathrm{d}W(t) \rightarrow v(t_2) - v(t_1) \sim N(0, \sigma^2(t_2 - t_1))$

$\mathrm{d}s(t) = v(t)\mathrm{d}t \rightarrow s(t) = \int_0^t v(\tau)\mathrm{d}\tau$

It can represent a **malfunctioning speed control** where speed cannot be controlled, or very strong influences from unpredictable effects such as wind gusts, or line resistances

2. Speed follows an Ornstein-Uhlenbeck process **(OU)**

$$\textbf{[OU]:} \quad \begin{cases} \mathrm{d}v(t) = \beta(v_{\text{CRUISE}} - v(t))\mathrm{d}t + \sigma \mathrm{d}W(t) \\ \mathrm{d}s(t) = v(t)\mathrm{d}t \end{cases}$$

$\rightarrow$ Mean-reverts to v_{CRUISE}

It can represent the process of a **human train driver** who knows the planned speed and continuously controls the train speed to be as close as possible

Stochastic process models (2/2): CIR and DMR

3. Cox-Ingersoll-Ross process (**CIR**)

[CIR]:
$$\begin{cases} \mathrm{d}v(t) = \beta(v_{\text{CRUISE}} - v(t))\mathrm{d}t + \widehat{\sigma}(v(t))\,\mathrm{d}W(t) \\ \mathrm{d}s(t) = v(t)\mathrm{d}t \end{cases}$$

$$\widehat{\sigma}(v) := \sigma\sqrt{\frac{v \cdot (v_{\text{MAX}} - v)}{v_{\text{CRUISE}} \cdot (v_{\text{MAX}} - v_{\text{CRUISE}})}}$$

Similar to OU but speed is bounded through a speed-dependent volatility

4. Doubly mean-reverting, doubly bounded process (**DMR**)

[DMR]:
$$\begin{cases} \mathrm{d}v(t) = [\beta(v_{\text{CRUISE}} - v(t)) + \alpha\,(v_{\text{CRUISE}}\,t - s(t))]\,\mathrm{d}t + \widehat{\sigma}(v(t))\,\mathrm{d}W(t) \\ \mathrm{d}s(t) = v(t)\mathrm{d}t \end{cases}$$

It can model how a **computer**, aware of precise position of current and ahead vehicle, can steer the system towards a desired space headway

Full driving dynamics and deterministic benchmark

- When a yellow signal is triggered, the train decelerates towards an approach speed
- **Full driving dynamics** combine a stochastic process with possible deceleration phases

We also study a **deterministic benchmark**

- The follower has a constant speed v_{FOLLOWER}
- If yellow signal, constant deceleration to the approach speed
- If approach speed reached, constant acceleration until v_{FOLLOWER}

Three cases:

- If $v_{\text{FOLLOWER}} < v_{\text{CRUISE}}$, the follower will indefinitely get further away from the leader, no trigger
- If $v_{\text{FOLLOWER}} = v_{\text{CRUISE}}$, leader and follower keep constant distance
- If $v_{\text{FOLLOWER}} > v_{\text{CRUISE}}$, the follower runs faster, reach the minimum safety distance, trigger a yellow signal, decelerate, accelerate again, trigger again, indefinitely oscillating over this cycle

Analytical approaches

We can study these processes with two approaches:

1. by adapting **theoretical results** on stochastic processes (fancy but very sophisticated)
2. by **Monte Carlo simulation** of multiple stochastic process trajectories (quite straightforward)

Why theory is complex?

1. It involves stochastic differential equations (due to stochastic processes)
2. It involves difficult stochastic differential equations (due to integrated stochastic processes)
3. It involves exogenous dynamics, e.g., the deceleration phase due to trigger

What can we study?

1. Simple processes/integrated processes (BM, OU)
2. Performance indicators independent of external dynamics, e.g., **first time to yellow**

First hitting time density (FHTD)

- Standard BM is the only case where a closed-form expression exists

$$p(t|v_0,\eta) = \frac{|\eta - v_0|}{\sqrt{2\sigma\pi t^3}} \exp\left(-\frac{(\eta - v_0)^2}{2\sigma^2 t}\right) \longrightarrow$$ Called inverse Gaussian distribution

- Standard OU: complex (only easy case is with threshold equal to mean-reversion)

- Integrated BM and OU: very complex. The FHTD satisfies

$$1-\mathrm{Erf}\left(\frac{\eta - \mathbf{m}_1(t|\mathbf{0},0)}{\sqrt{2Q_{11}(t|0)}}\right) = \int_0^t p(\vartheta|\mathbf{0},0)\mathbb{E}_{Z(\vartheta)}\left[1 - \mathrm{Erf}\left(\frac{\eta - \mathbf{m}_1(t|[\eta, X_2(\vartheta)],\vartheta)}{\sqrt{2Q_{11}(t|\vartheta)}}\right)\right] \mathrm{d}\vartheta$$ where

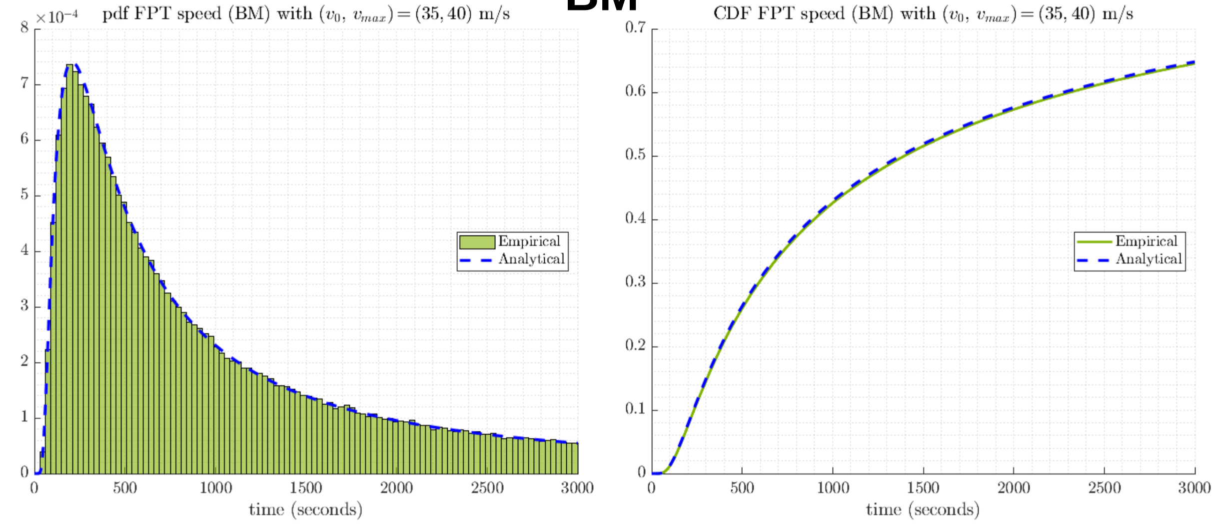
$$\mathrm{Erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}dt, \qquad \mathrm{d}\mathbf{X}(t) = [A\mathbf{X}(t) + \mathbf{M}]\mathrm{d}(t) + G\mathrm{d}\mathbf{W}(t),$$

$$\mathbf{m}(t|\mathbf{y},t_0) = e^{A(t-t_0)}\left[\mathbf{y} + \int_{t_0}^t e^{-A(u-t_0)}\mathbf{M}\mathrm{d}u\right], \quad Q(t|t_0) = \int_{t_0}^t e^{A(t-u)}GG^\intercal e^{A(t-u)}du$$

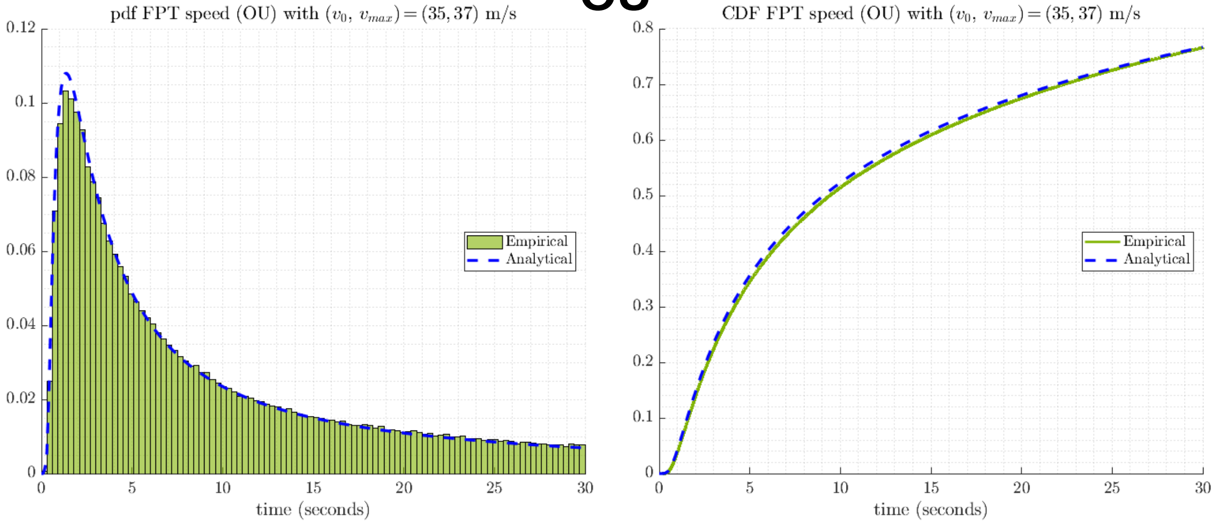
Which is approximated by solving a system of linear equations

FHTD: Analytical vs. simulation

BM



OU



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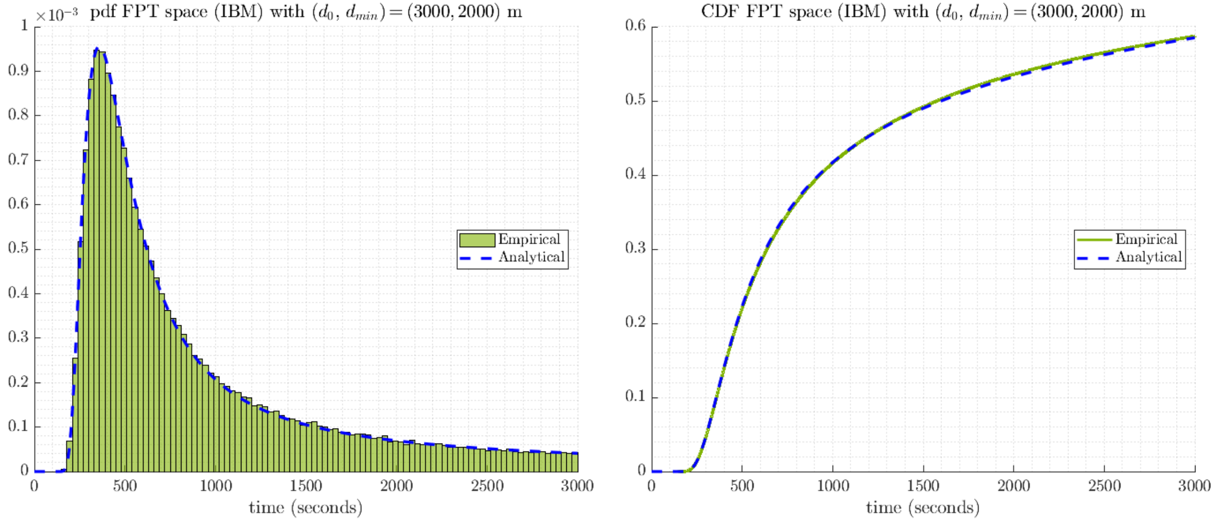
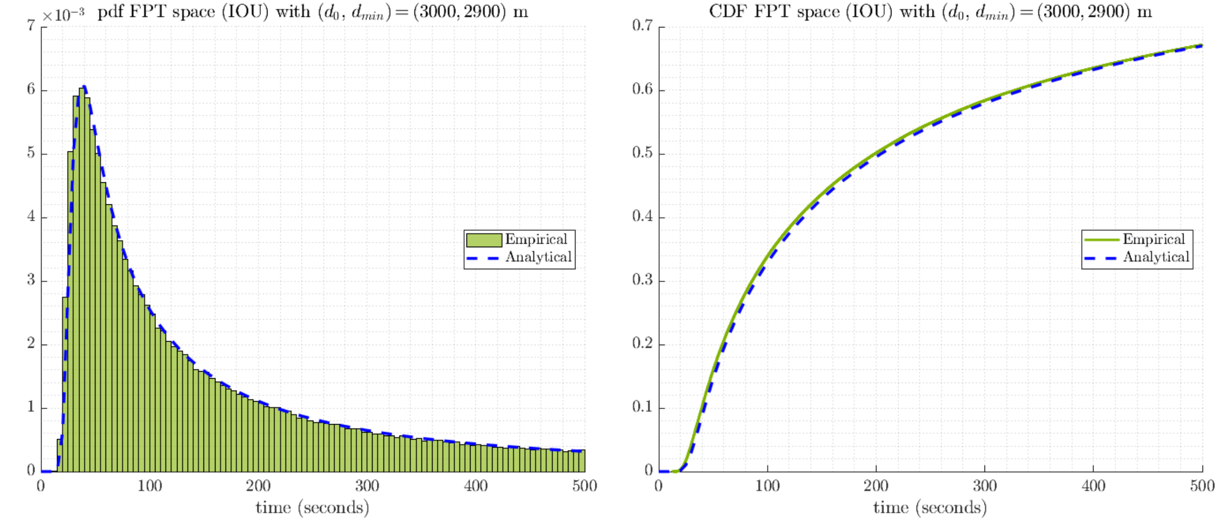


IOU



Analytical vs. simulation: takeway

- We were able to determine the FHTD of some processes **analytically:**
  - The analytical results match with our simulation
  - This suggests that our **simulation tool is accurate** (despite known issues of simulation)

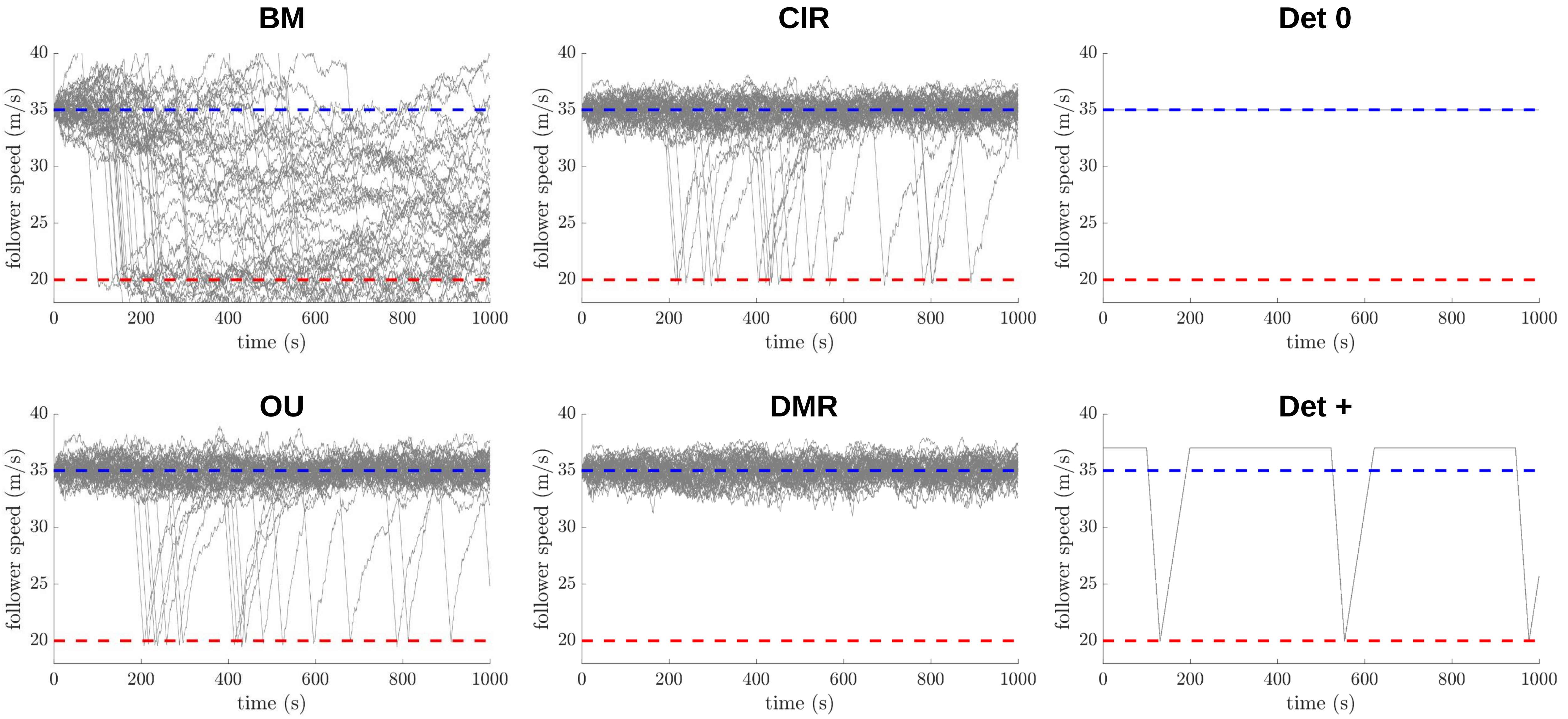
- **In general, we have to rely on simulation:**
  - For more complex stochastic processes (CIR, DMR and their integrals)
  - For performance indicators that require the full dynamics

- We show next results from simulation, where:

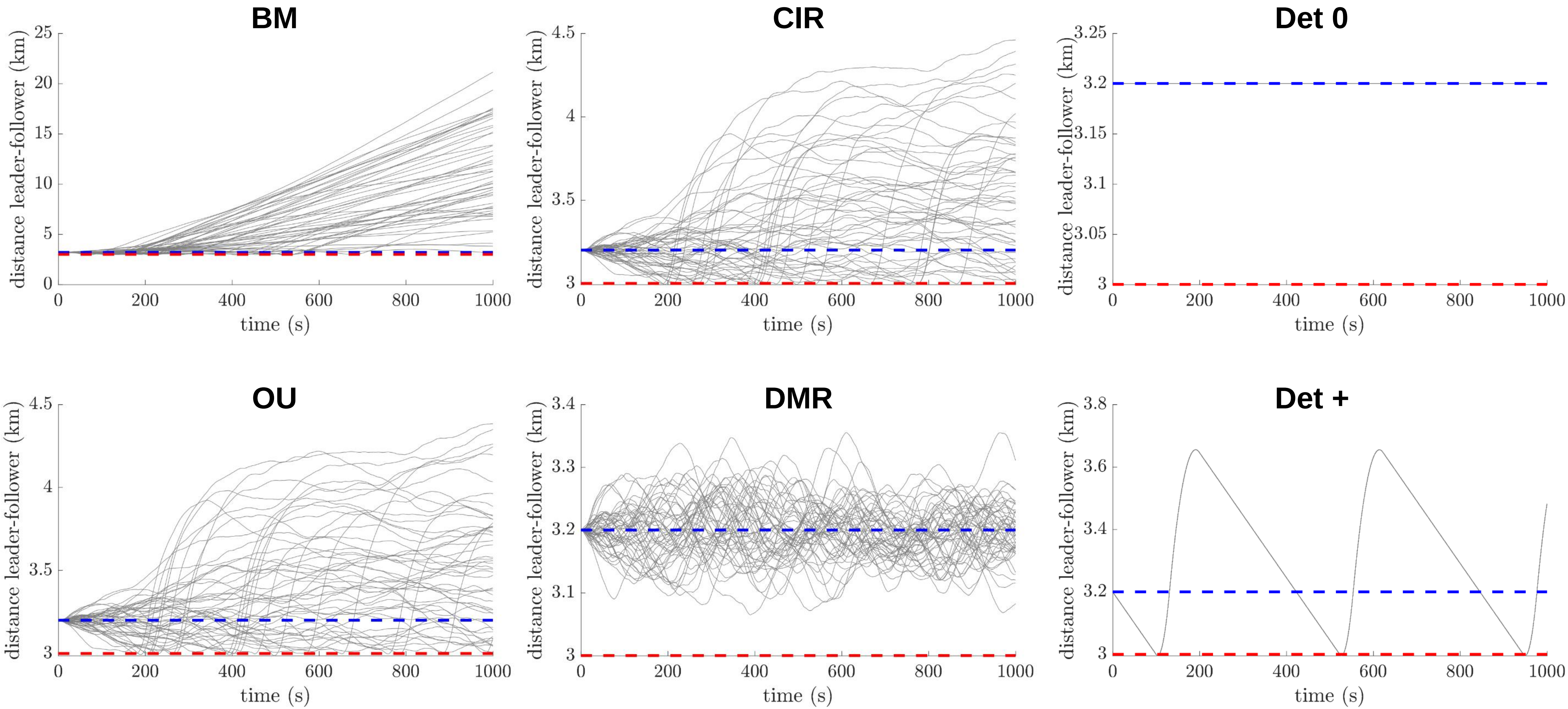
$v(0) = v_{\text{CRUISE}} = 35\,\text{m/s}$ $\quad v_{\text{MAX}} = 40\,\text{m/s}$ $\quad d_{\text{MIN}} = 3\,\text{km}$ $\quad \beta = 0.02$

$v_{\text{APPROACH}} = 20\,\text{m/s}$ $\quad d_0 = 3.2\,\text{km}$ $\quad \sigma = 0.2$ $\quad \alpha = 0.0005$

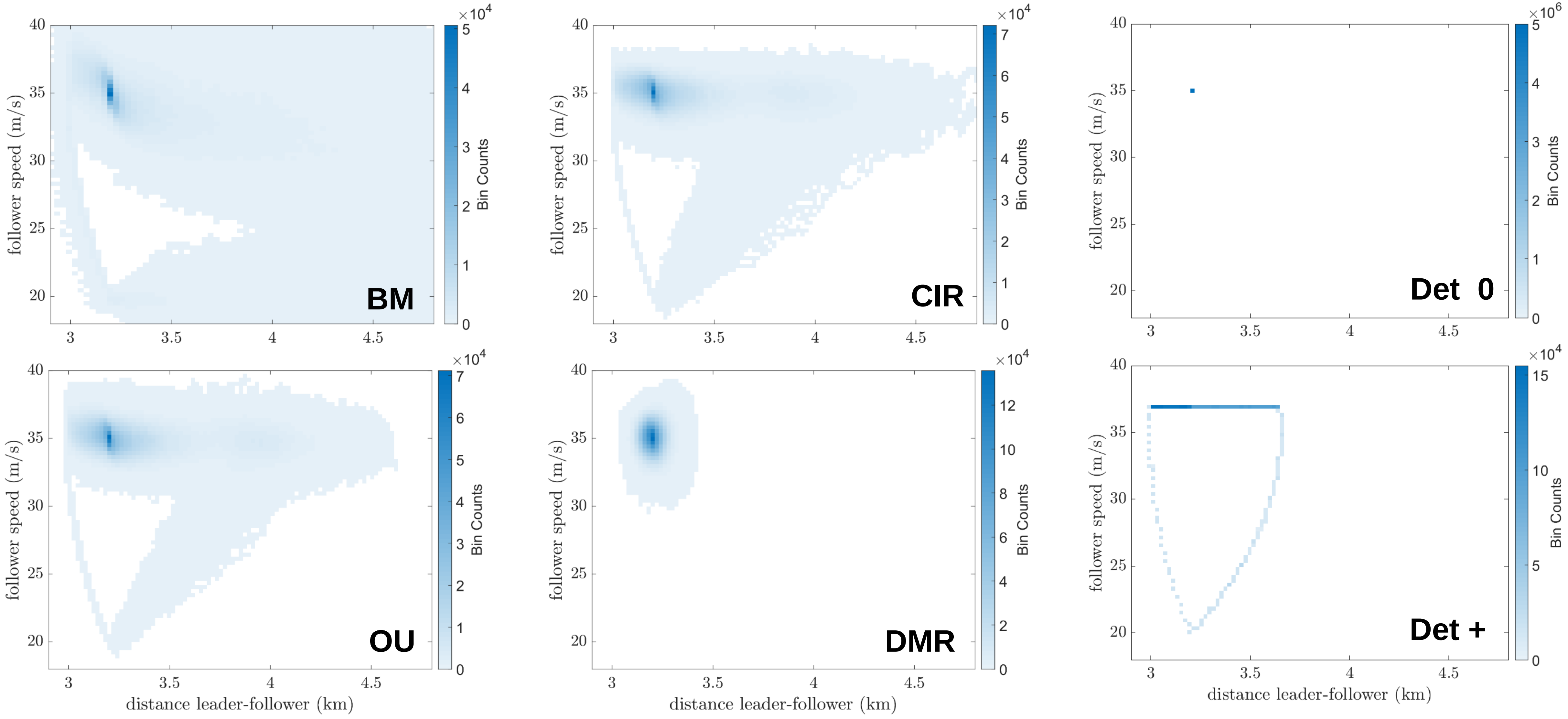
Time-speed trajectories



Time-space trajectories



Space-speed trajectories



Performance indicators

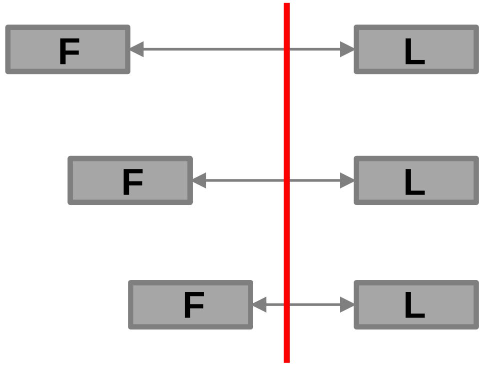
Table 1: Analysis of aggregate properties from the four stochastic process models (horizon 1 hour).

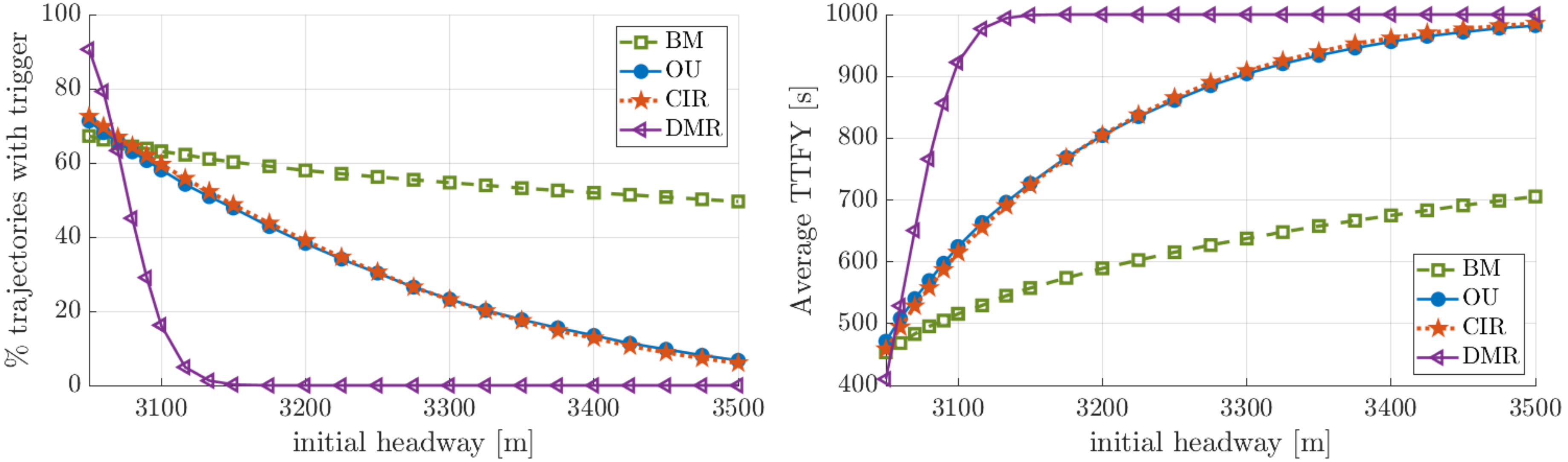
Performance indicator		Unit	BM	OU	CIR	DMR	DET_0	DET_+
Trajectories with at least one yellow signal		[%]	70.4	65.2	65.9	0.0	0.0	100.0
Yellow signals per 1000 seconds		[-]	0.20	0.19	0.19	0.00	0.00	2.50
Time to first yellow	average	[s]	1474	1962	1941	>3600	>3600	105
	50th percentile	[s]	536	1627	1563	>3600	>3600	105
	5th percentile	[s]	104	214	230	>3600	>3600	105
Space headway	average	[km]	20.25	3.66	3.66	3.20	3.20	3.33
	50th percentile	[km]	15.14	3.62	3.62	3.20	3.20	3.33
	95th percentile	[km]	55.24	4.41	4.43	3.28	3.20	3.64
Speed follower	average	[m/s]	24.24	34.81	34.81	35.00	35.00	34.88
	50th percentile	[m/s]	24.19	34.94	35.00	35.03	35.00	37.00
	95th percentile	[m/s]	37.08	36.60	36.51	36.59	35.00	37.00

System throughput (vehicles/hour) → **15.8** **34.2** **34.2** **39.8** **42.0**

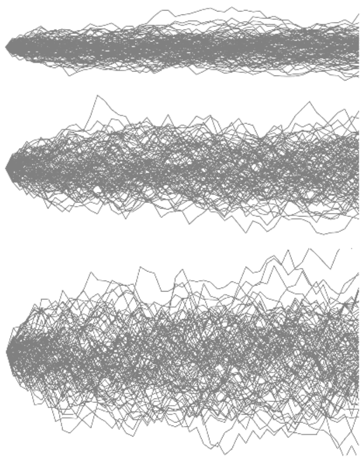
Sensitivity analysis

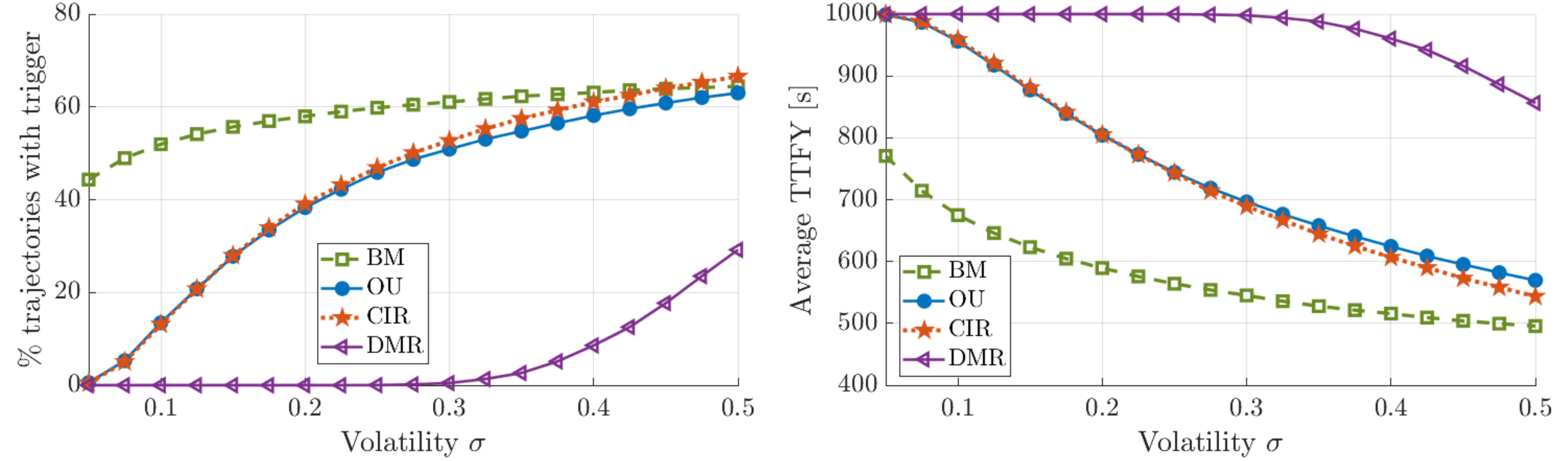
- Initial headway





- Process volatility





Conclusion

- We developed a novel railway traffic flow model
- We proposed four stochastic processes that can model different driving situations (e.g., human driver and ATO)

- We quantified the benefit of ATO in terms of added regularity and reliability
- However, current trains do not know about the distance of the traffic flow ahead, nor their speed; The only immediate information they have is whether there is yellow signal, or not

Future work includes

- Generalization and identification of analytic similarities with bus bunching
- Identification/calibration of suitable parameters for the real-life processes

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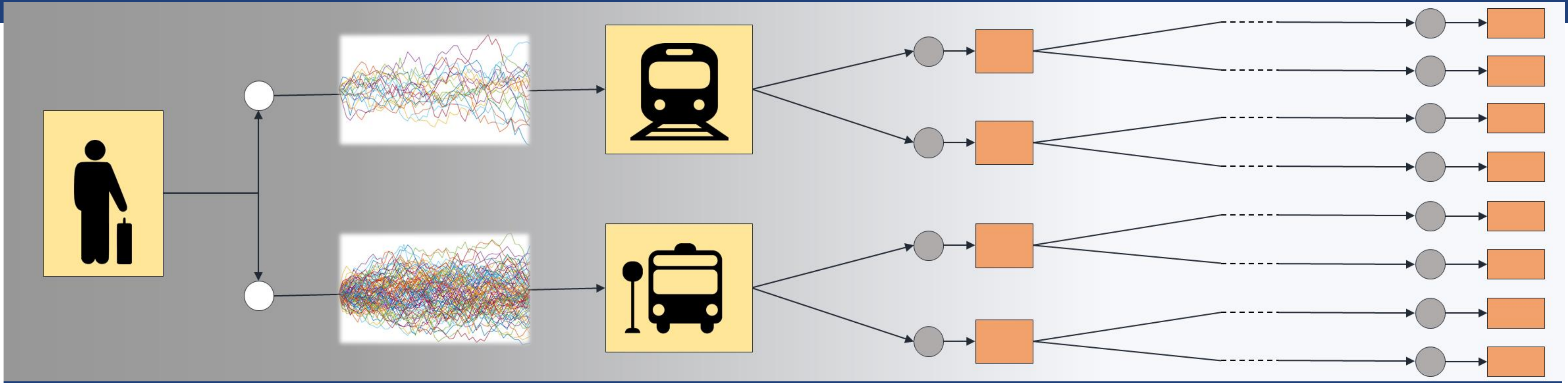

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