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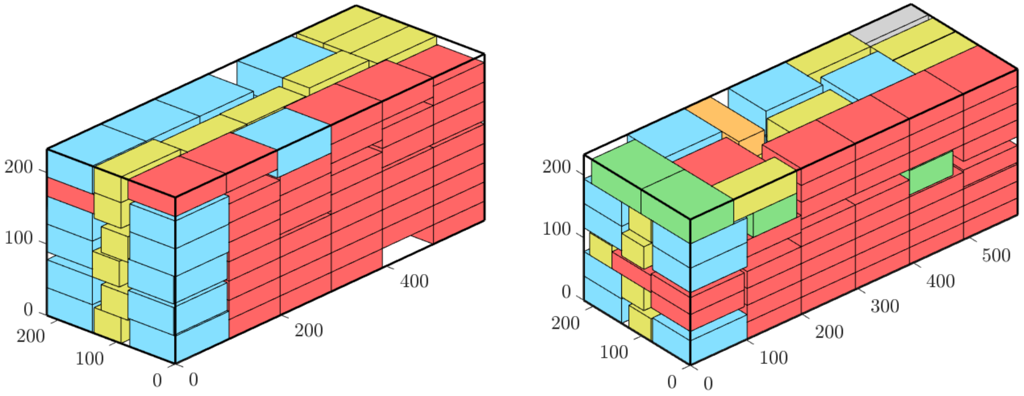

Modeling soft unloading constraints in the multi-drop CLP

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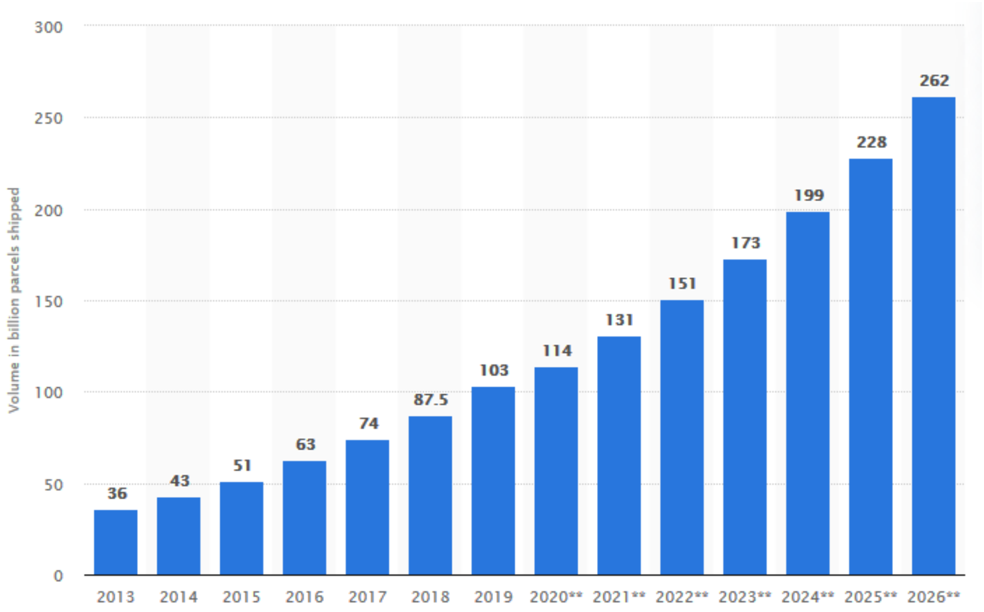
Motivation

- The logistics sector is growing
- The number of parcels shipped globally is expected to **double** from 2020 (>100 billion) to 2025 (>200 billion)
- **Covid-19** has exacerbated this trend



Global parcel shipping volume (billion parcels); statista.com (2020)

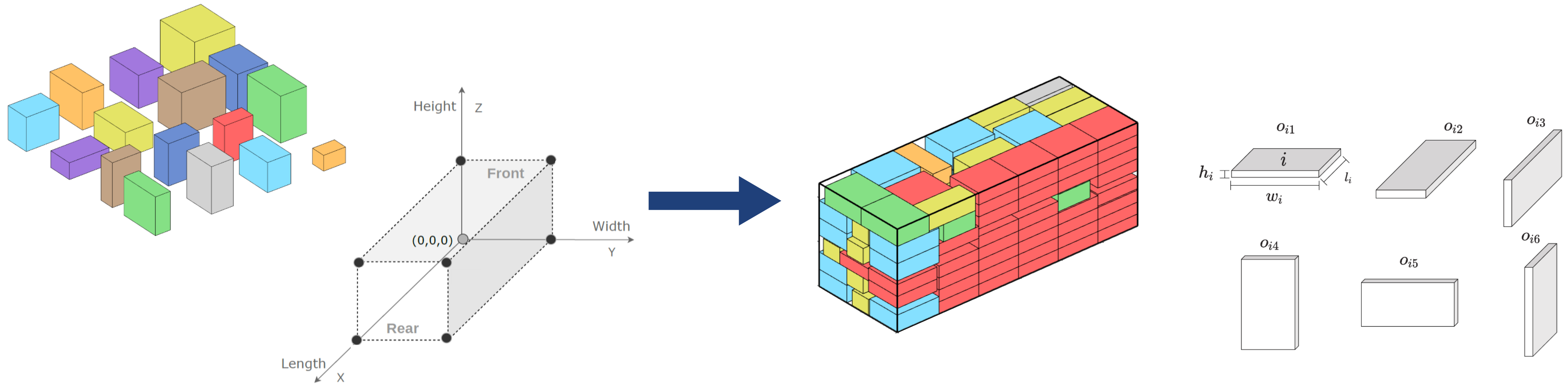
- This growth is an **opportunity** for logistics operators…

  …but also a **challenge**, due to increasing operating costs and environmental concerns

- The **efficient loading of vehicles / containers** plays a key role in turning this opportunity into higher profits / sustainability, but is often a complex task in practice

Container Loading Problem (CLP)

- Given a set of 3D items (cuboids), each with a value,
- and a larger container,
- load items maximizing value, s.t.: non-overlapping, boundaries, orthogonal placements



- CLP is well studied, including practical constraints (e.g., Bortfeldt and Wascher 2013, Silva et al. 2018, Alonso et al. 2019, Gajda et al. 2021, Nascimento et al. 2021)

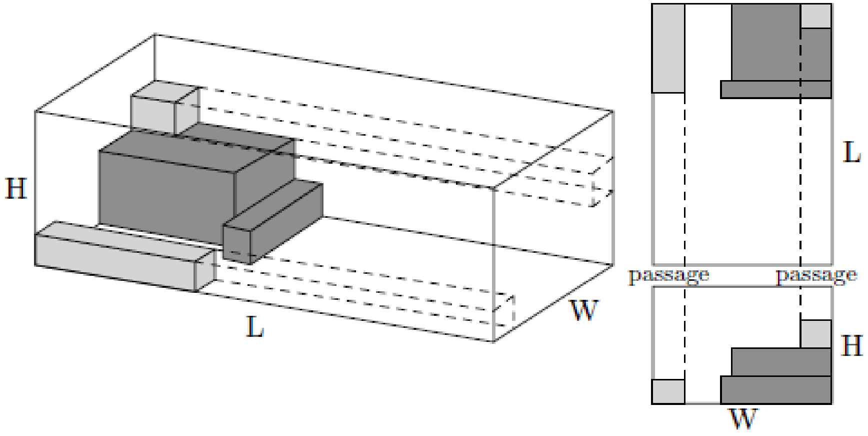
Multi-drop container loading problem (MDCLP)

- Multiple customers to visit in a predetermined sequence (no routing)
- Account for the unloading sequence to avoid relocating cargo during delivery
- Also well studied but **unloading constraints** treated as **hard constraints** (Gendreau et al. 2006, Iori et al. 2007, Christensen and Rouse 2009, Pan et al. 2009, Fuellerer et al. 2010, Iori and Martello 2010, Liu et al. 2011, Junqueira et al. 2012, de Queiroz and Miyazawa 2013, Martinez et al. 2015, Hokama et al. 2016, Pollaris et al. 2016, Iori et al. 2020, Ferreira et al. 2021, Nascimento et al. 2021)

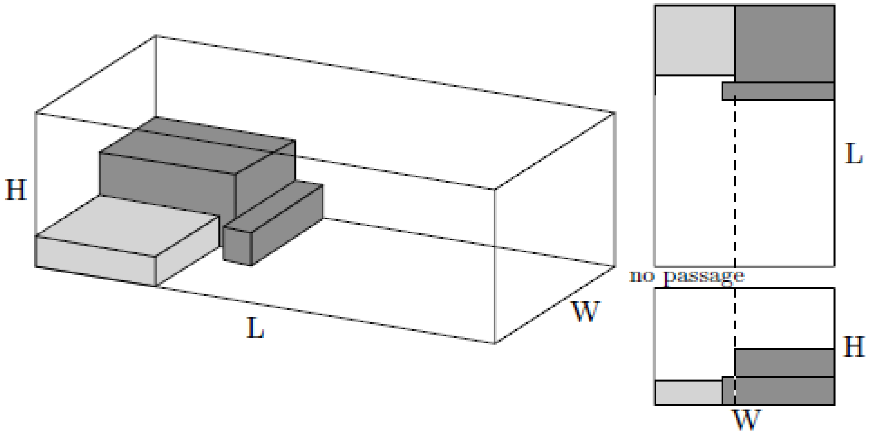
  → very constrained loading solutions that carry less cargo value

- Our goal: use **soft unloading constraints** to manage the **trade-off between cargo value and penalties** due to constraint violations
- Literature:
  - Gajda et al. (2021): count number of relocations and do not minimize it explicitly in the algorithm
  - Lurkin and Schyns (2015): aircraft loading problem with fixed slots arranged into rows

Visibility and Above constraints



(a) Visible



(b) Not visible

Figure source: Martinez et al. 2015

- **Visibility**: item to unload visible from unloading side

  Visibility is violated when

  $$(c_i < c_j) \wedge (x_j \geq x_i + l_i) \wedge \textsc{Overlap}(i,j,y) \wedge \textsc{Overlap}(i,j,z)$$

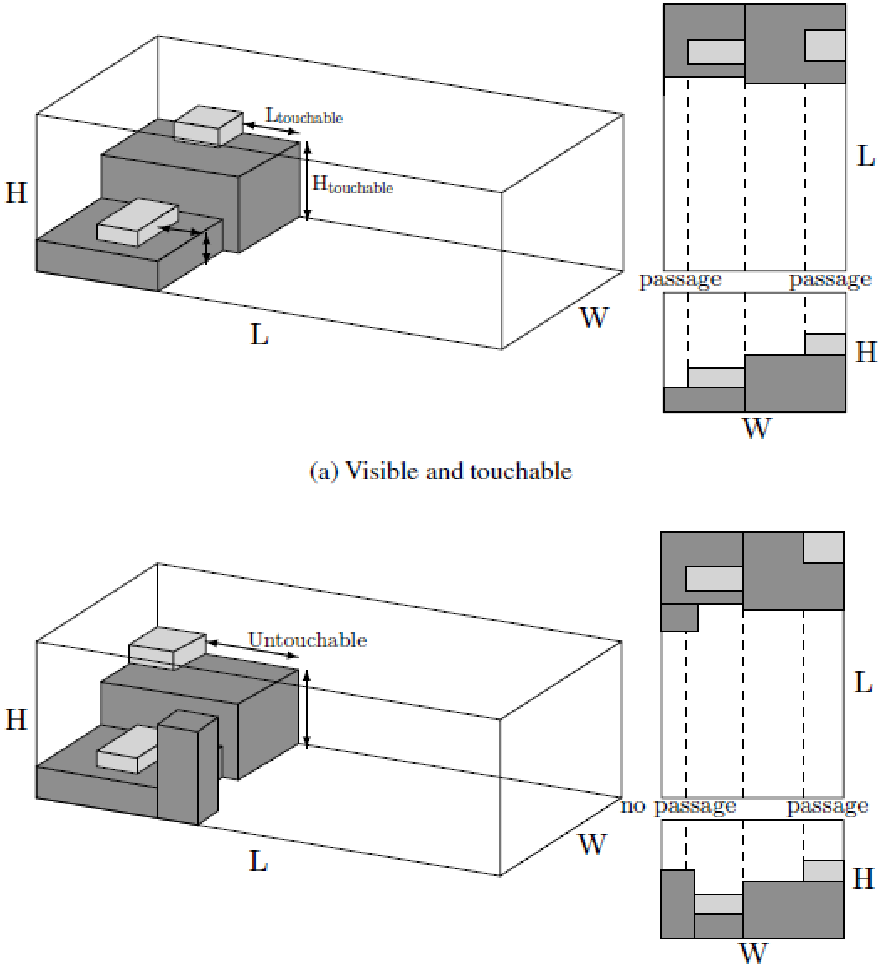
  where

  $$\textsc{Overlap}(i,j,x) \iff (x_i \leq x_j < x_i + l_i) \vee (x_j \leq x_i < x_j + l_j)$$

  We assign a cost / penalty $f_{\mathrm{V}}(q_j, v_j, z_j) \geq 0$

- **Above**: item to unload is not under other items.
  Similar formulation, penalty $f_{\mathrm{A}}(q_j, v_j, z_j) \geq 0$

Reachability constraints



(a) Visible and touchable

(b) Not visible or not touchable

Figure source: Martinez et al. 2015

- Item to unload is unreachable by a human operator (or machine / forklift)
- Distance between item and position the operator can reach is larger than a fixed quantity

$$(c_i < c_j) \wedge (x_j + l_j - (x_i + l_i) \geq \min\{H_{touchable} - z_i, L_{touchable}\})$$

- Penalty for violations $f_{\mathrm{R}}(q_j, v_j, z_j, \delta_{ij})$

MIP formulation (1/2)

- Above + Visibility
- Rotations omitted here for simplicity (full model in appendix)

1 Boundaries

$$t_i,\ b_{ij},\ f_{ij},\ u_{ij}, \in \{0,1\} \quad \forall i,j \in \mathcal{B},$$
$$x_i \in [0, L - l_i], \quad \forall i \in \mathcal{B},$$
$$y_i \in [0, W - w_i], \quad \forall i \in \mathcal{B},$$
$$z_i \in [0, H - h_i], \quad \forall i \in \mathcal{B}.$$

2 Non-overlapping constraints

$$b_{ij} + b_{ji} + f_{ij} + f_{ji} + u_{ij} + u_{ji} + (1 - t_i) + (1 - t_j) \geq 1 \qquad \forall i,j \in \mathcal{B},\ i < j,$$
$$x_i + l_i \leq x_j + L(1 - b_{ij}) \qquad \forall i,j \in \mathcal{B},$$
$$y_i + w_i \leq y_j + W(1 - f_{ij}) \qquad \forall i,j \in \mathcal{B},$$
$$z_i + h_i \leq z_j + H(1 - u_{ij}) \qquad \forall i,j \in \mathcal{B}.$$

3 Track when items do overlap

$$x_j \leq x_i + l_i + L\, b_{ij} \qquad \forall i,j \in \mathcal{B},\ c_i \neq c_j,$$
$$y_j \leq y_i + w_i + W\, f_{ij} \qquad \forall i,j \in \mathcal{B},\ c_i \neq c_j,$$
$$z_j \leq z_i + h_i + H\, u_{ij} \qquad \forall i,j \in \mathcal{B},\ c_i \neq c_j.$$

MIP formulation (2/2)

4 Model joint overlap in two dimensions (requires new binaries and constraints)

$$a_{ij} = 1 \iff \textsc{Overlap}(i, j, x) \wedge \textsc{Overlap}(i, j, y)$$

$$d_{ij} = 1 \iff \textsc{Overlap}(i, j, y) \wedge \textsc{Overlap}(i, j, z)$$

$$\begin{aligned}
&b_{ij} + b_{ji} + f_{ij} + f_{ji} \geq 1 - a_{ij} && \forall i, j \in \mathcal{B},\, c_i < c_j,\\
&b_{ij} + b_{ji} + f_{ij} + f_{ji} \leq 2(1 - a_{ij}) && \forall i, j \in \mathcal{B},\, c_i < c_j,\\
&f_{ij} + f_{ji} + u_{ij} + u_{ji} \geq 1 - d_{ij} && \forall i, j \in \mathcal{B},\, c_i < c_j,\\
&f_{ij} + f_{ji} + u_{ij} + u_{ji} \leq 2(1 - d_{ij}) && \forall i, j \in \mathcal{B},\, c_i < c_j.
\end{aligned}$$

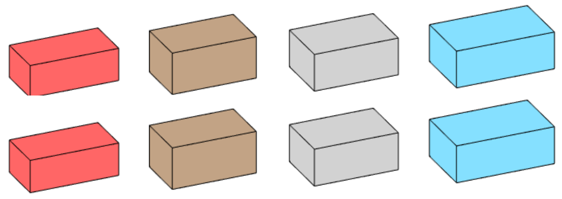
5 Couple overlapping in two directions with condition on third direction

$$\begin{aligned}
&p_{ij} + 1 \geq a_{ij} + u_{ij} && \forall i, j \in \mathcal{B},\, c_i < c_j,\\
&r_{ij} + 1 \geq d_{ij} + b_{ij} && \forall i, j \in \mathcal{B},\, c_i < c_j.
\end{aligned}$$

6 Objective: $\max \sum_{i \in \mathcal{B}} \pi_i t_i - \sum_{i \in \mathcal{B}} \sum_{j \in \mathcal{B}: c_i < c_j} \left[p_{ij} \cdot f_{\mathrm{A}}(q_j, v_j, z_j) + r_{ij} \cdot f_{\mathrm{V}}(q_j, v_j, z_j) \right]$

Illustrative example: Setting

- Small instance: 4 customers, 8 items, all rotations allowed



Demand	Length (l_i, cm)	Width (w_i, cm)	Height (h_i, cm)	Volume (v_i, m^3)	Weight (q_i, ton)	Value (π_i)	customer (c_i)
2	95	50	35	0.166	0.22	0.22	1
2	90	55	45	0.223	0.24	0.24	2
2	90	60	40	0.216	0.26	0.26	3
2	105	65	40	0.273	0.28	0.28	4

- Linear penalties in volume / weight (i.e., MILP):

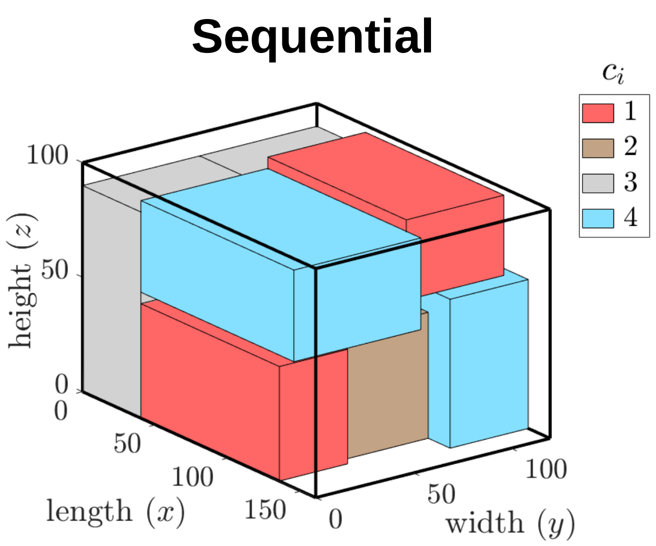
$$f_{\mathrm{A}}(q_j, v_j, z_j) := \alpha_{\mathrm{A}} q_j + \beta_{\mathrm{A}} v_j + \gamma_{\mathrm{A}},$$
$$f_{\mathrm{V}}(q_j, v_j, z_j) := \alpha_{\mathrm{V}} q_j + \beta_{\mathrm{V}} v_j + \gamma_{\mathrm{V}},$$

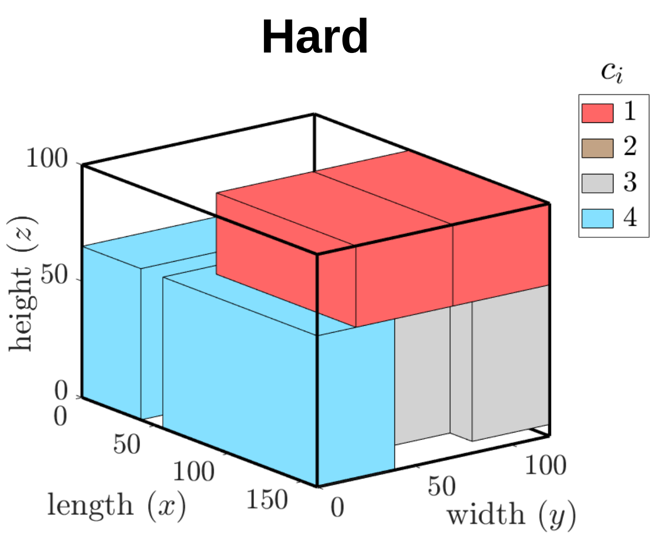
- With coefficients $(\alpha_{\mathrm{A}}, \beta_{\mathrm{A}}, \gamma_{\mathrm{A}}, \alpha_{\mathrm{V}}, \beta_{\mathrm{V}}, \gamma_{\mathrm{V}}) = (0.1, 0.1, 0, 0.1, 0.1, 0)$

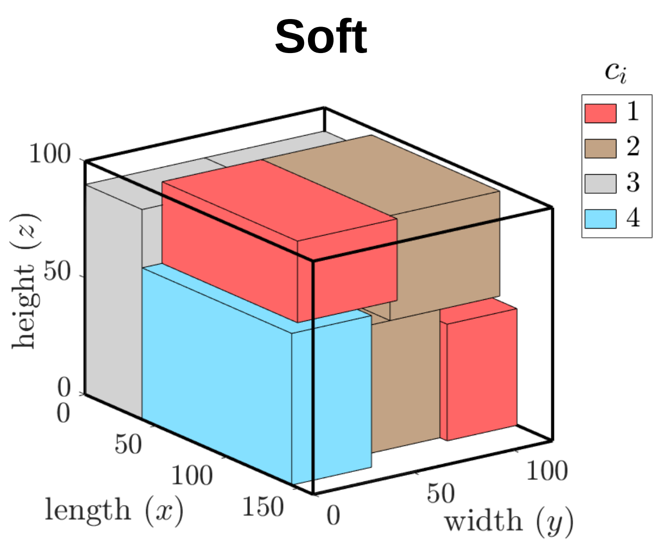
- Solution approach / Model:

Sequential strategy | **Hard unloading constraints** | **Soft unloading constraints**

Illustrative example: Results



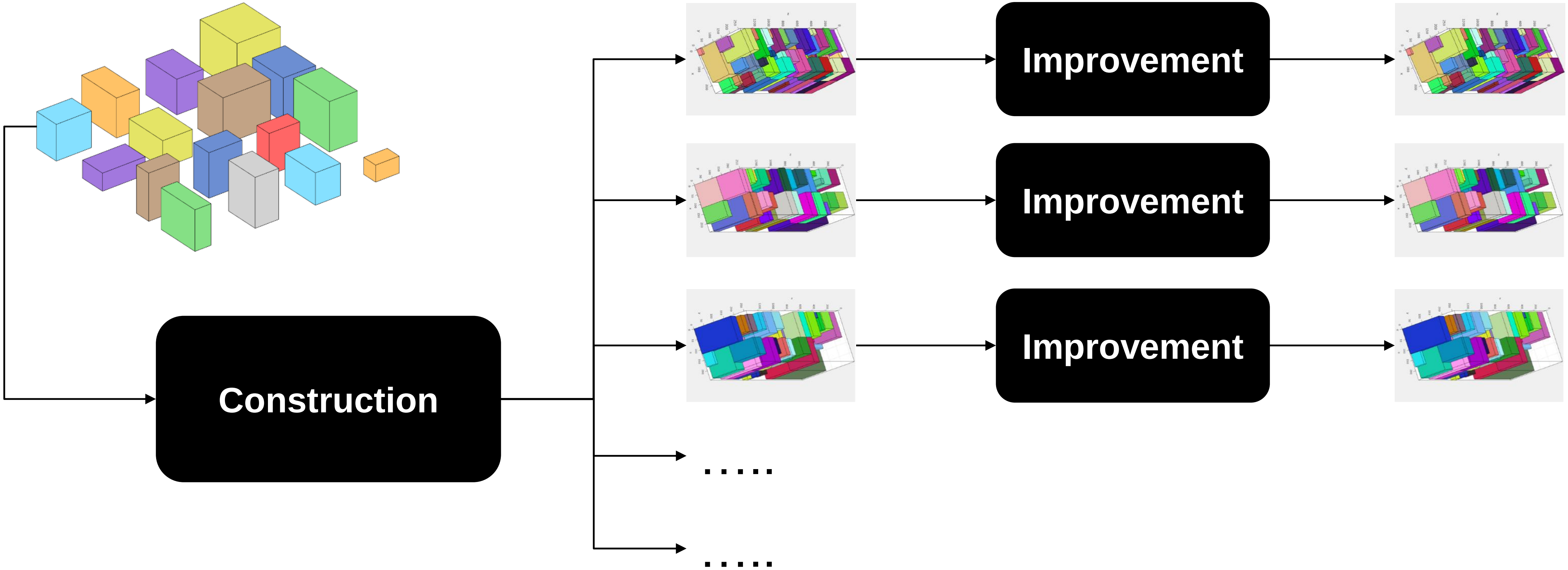




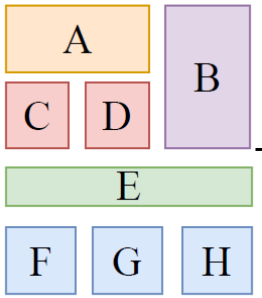
Strategy	Objective	Cargo		Penalty		Runtime (s)
		Value	# items	Amount	# violations	
Sequential	74.2	**88.0**	7	13.8	5	0.8
Hard	76.0	76.0	6	**0.0**	0	8.6
Soft	**80.9**	86.0	7	5.1	2	22.1

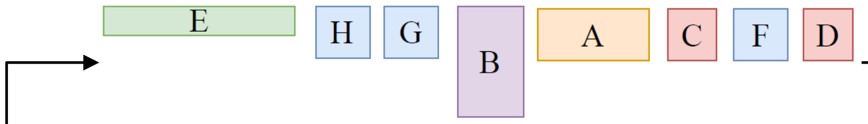
Heuristic framework

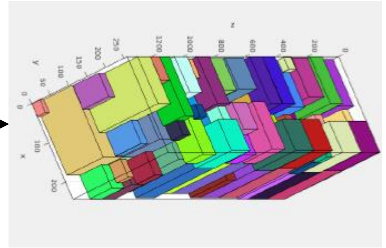
- Motivation: **1** larger instances **2** general penalties **3** reachability constraints



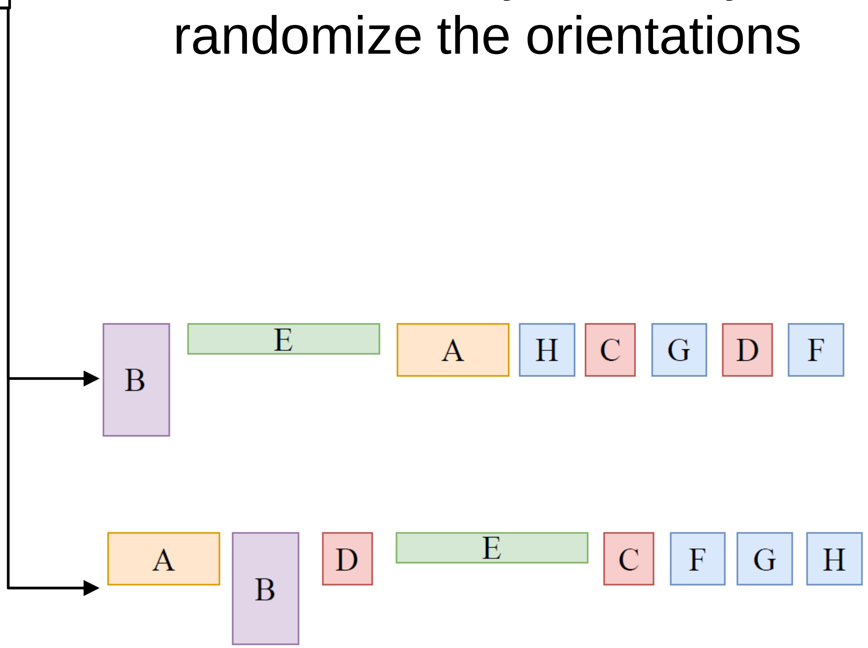
Construction Phase



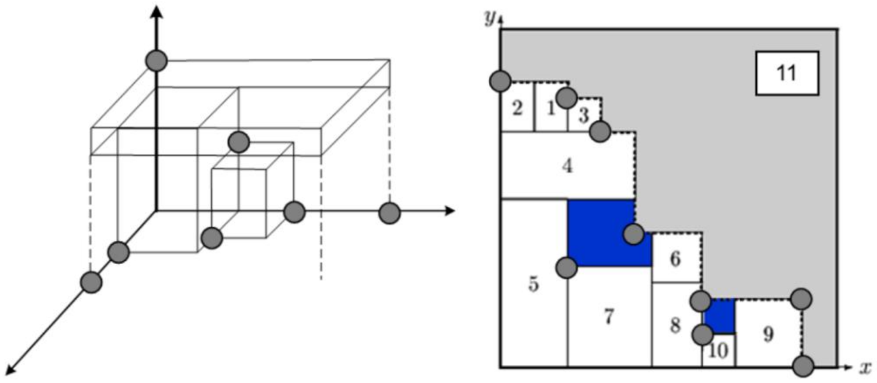




- **Sort** (e.g., based on volume, area, customer number etc.)
- **Randomize** by similarity and randomize the orientations



- Fill container with Extreme Points heuristic (Crainic et al. 2008)

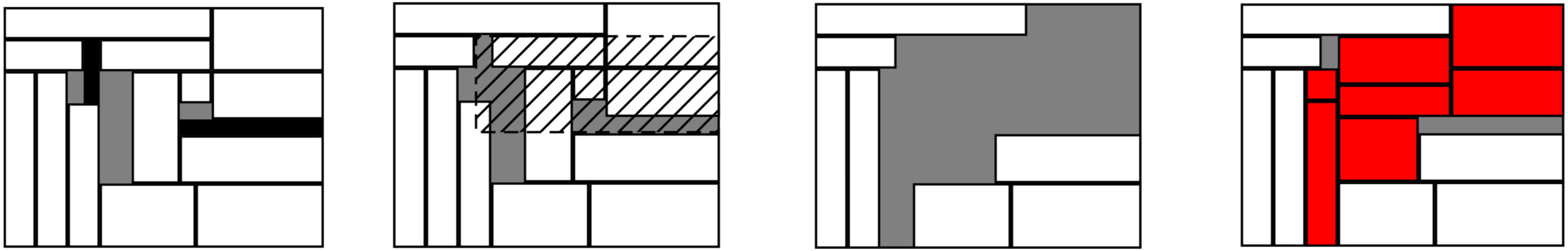


(a) (b)

- Re-attempt loading with a retry list

Improvement phase (1/3)

- Unloading constraints are considered only implicitly during construction
- Idea: Improve objective by iteratively emptying and reconstructing regions



(a) Selecting the spaces (b) Defining the region (c) Removing overlapping boxes (d) Filling

Figure source: Parreño et al. 2010

Need to specify:

- How to select two items defining the region **(a)** → Use penalty information
- How to refill the region **(d)** → Best cargo value/penalty trade-off

Improvement phase (2/3)

- Choose item pair (and corresponding region) according to three different approaches

M-1	Total penalty
M-2	Penalty density
M-3	Penalty density and empty space density

- To fill region, choose item and orientation using best-fit decreasing with merit function

$$m_{io} := \pi_i - \left\{ \sum (C_{i^o j} + C_{j i^o}) : j \in \mathcal{S} \right\}$$

where $C_{i^o j}$ is the conflict between item i with orientation o and item j, and $\mathcal{S}$ is the solution

Improvement phase (3/3)

- After re-construction, we:
  - compress the solution into a “gapless” packing
  - and refill the container once more

- We use the interval graphs (Fekete et al. 2007) to shift items towards lower coordinates

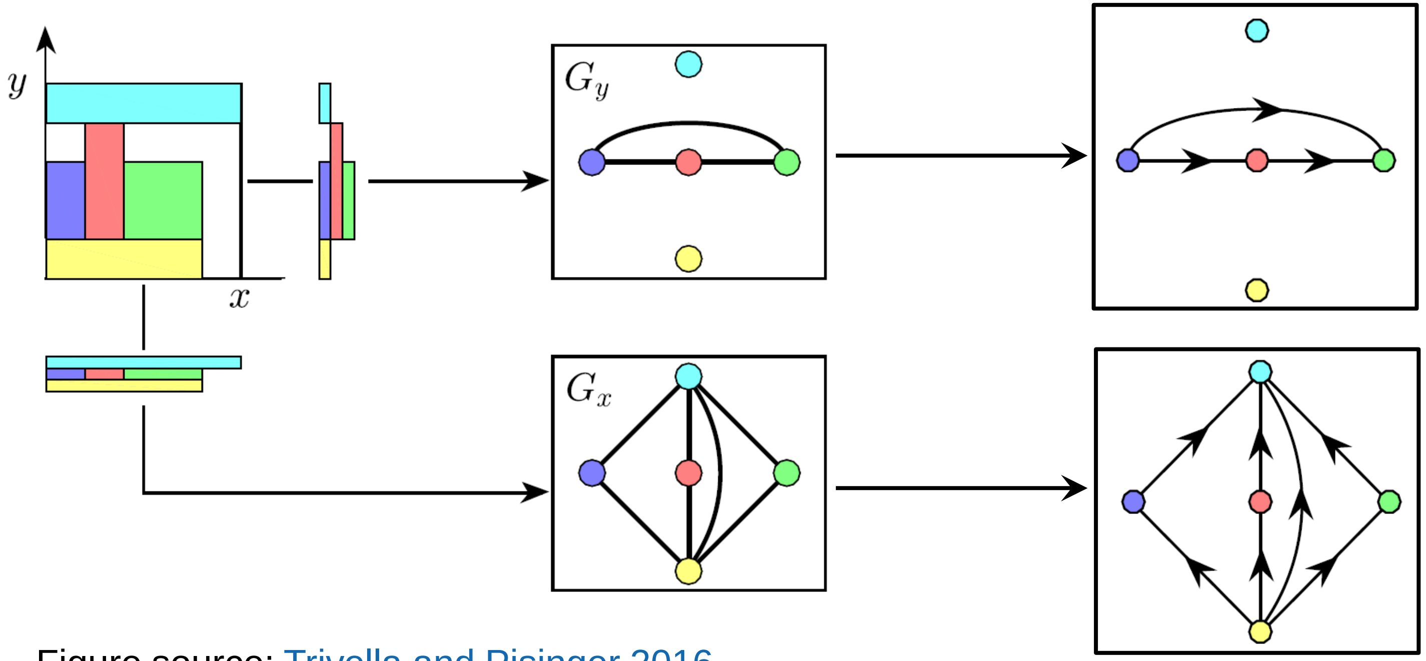


Figure source: Trivella and Pisinger 2016

- Pick best solution among:
  - Initial
  - Reconstructed
  - Compressed and refilled

Numerical study

- Instances:
  - BR instances from literature (Bischoff et al. 1995, Davies and Bischoff 1999)
  - 1500 instances divided in 15 classes; average of 178 items, maximum of 1961 items
  - We assign weight, value, and divide items among 2 to 8 customers

- Above + Visibility + Reachability:

$$f_{\mathrm{A}}(q_j, v_j, z_j) = (1 + \gamma z_j) \cdot (\alpha q_j + \beta v_j)$$
$$f_{\mathrm{V}}(q_j, v_j, z_j) = (1 + \gamma z_j) \cdot (\alpha q_j + \beta v_j)$$
$$f_{\mathrm{R}}(q_j, v_j, z_j, \delta_{ij}) = (1 + \gamma \delta_{ij}) \cdot (\alpha q_j + \beta v_j)$$

Set	Penalty level	α	β	γ
P-1	low	$2.5 \cdot 10^{-3}$	$5.0 \cdot 10^{-3}$	$4.5 \cdot 10^{-3}$
P-2	medium	$7.5 \cdot 10^{-3}$	$15.0 \cdot 10^{-3}$	$4.5 \cdot 10^{-3}$
P-3	high	$12.5 \cdot 10^{-3}$	$25.0 \cdot 10^{-3}$	$4.5 \cdot 10^{-3}$
P-4	very high	$17.5 \cdot 10^{-3}$	$35.0 \cdot 10^{-3}$	$4.5 \cdot 10^{-3}$

- Solution approach:

Sequential strategy	**Hard unloading constraints**	**Soft unloading constraints**
Only construction phase (30s)	Modified construction phase (30s)	Construction (30s) and improvement of 5 solutions (20s each)

Comparison Hard vs. Soft

Objective value — Average improvement as percentage

Class	Hard	Soft				Δ (%)			
	P-*	P-1	P-2	P-3	P-4	P-1	P-2	P-3	P-4
BR1	77.1	84.1	81.4	80.4	79.9	9.0	5.5	4.2	3.6
BR2	76.2	83.4	80.6	79.4	78.7	9.4	5.8	4.2	3.3
BR3	72.8	80.9	77.0	75.3	74.5	11.2	5.8	3.5	2.4
BR4	72.4	80.4	76.5	75.0	74.4	11.0	5.6	3.6	2.6
BR5	72.1	79.6	75.6	74.0	73.2	10.4	4.8	2.7	1.6
BR6	70.7	79.0	74.6	72.8	72.0	11.7	5.5	3.0	1.8
BR7	70.0	78.1	73.7	72.3	71.1	11.6	5.3	3.3	1.7
BR8	69.4	77.7	73.4	71.4	70.4	12.0	5.9	3.0	1.6
BR9	69.4	77.0	72.7	70.9	70.0	11.0	4.7	2.2	1.0
BR10	68.9	76.5	72.2	70.4	69.4	11.0	4.8	2.1	0.8
BR11	69.3	76.4	72.4	70.4	69.8	10.4	4.5	1.7	0.7
BR12	68.4	76.1	71.4	69.6	68.8	11.2	4.4	1.6	0.6
BR13	68.9	76.3	72.0	70.0	69.2	10.7	4.4	1.6	0.4
BR14	68.3	75.8	71.4	69.4	68.6	11.1	4.5	1.6	0.4
BR15	68.7	75.9	71.4	69.6	68.9	10.5	4.0	1.3	0.3
Mean	70.8	78.5	74.4	72.7	71.9	10.8	5.0	2.6	1.5

- Improvement ranges in **0-12%**, with an average of **5%**
- **Higher** improvement levels achieved under **lower** penalties
- Exploiting the flexibility of soft unloading constraints is:
  - challenging under high and very high penalties
  - useful in general

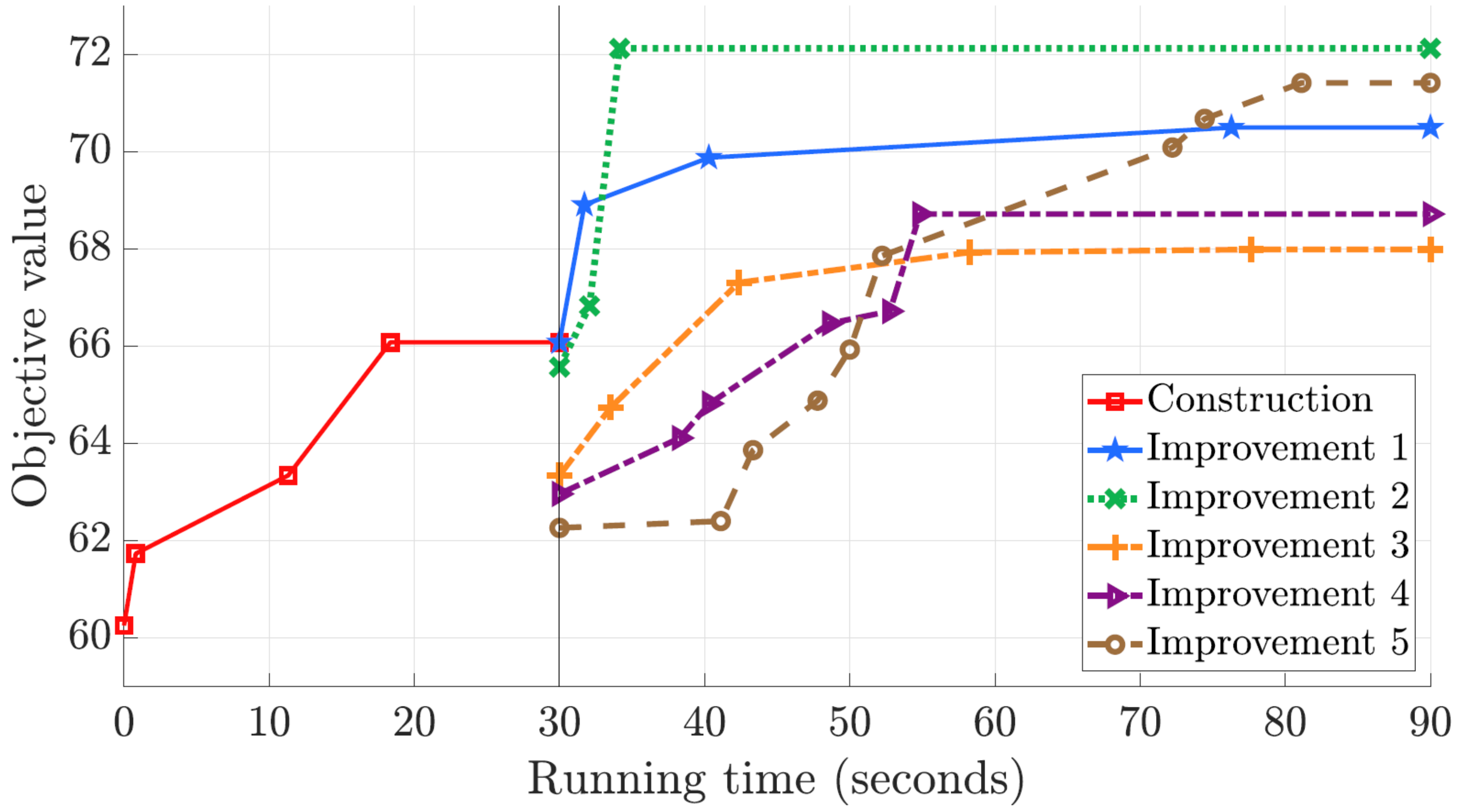
Comparison Sequential vs. Soft

Objective value → Sequential; Average improvement percentage → Δ (%)

Class	Sequential				Δ (%)			
	P-1	P-2	P-3	P-4	P-1	P-2	P-3	P-4
BR1	82.0	77.1	74.1	71.5	2.5	5.6	8.4	11.7
BR2	81.9	77.7	74.5	71.6	1.8	3.7	6.7	9.9
BR3	79.6	73.8	69.6	65.8	1.7	4.3	8.1	13.2
BR4	78.7	73.4	69.8	66.6	2.1	4.2	7.4	11.7
BR5	78.2	72.0	67.7	63.8	1.8	5.0	9.4	14.9
BR6	77.3	71.8	67.9	64.2	2.2	3.9	7.4	12.2
BR7	76.3	71.1	67.1	63.5	2.3	3.7	7.6	12.0
BR8	75.7	70.7	67.3	64.2	2.7	3.8	6.1	9.7
BR9	75.1	70.6	67.3	64.2	2.5	3.0	5.3	9.0
BR10	74.4	70.2	66.9	63.8	2.7	2.9	5.2	8.8
BR11	74.5	70.2	67.1	64.2	2.6	3.0	4.9	8.8
BR12	74.1	69.6	66.2	62.9	2.7	2.6	5.1	9.3
BR13	74.4	70.4	67.3	64.3	2.6	2.3	4.1	7.6
BR14	74.0	69.6	66.1	62.8	2.4	2.6	5.0	9.2
BR15	73.8	69.5	66.0	62.8	2.8	2.8	5.3	9.8
Mean	76.7	71.8	68.3	65.1	2.4	3.5	6.4	10.5

- Improvement ranges in **2-15%**, with an average of **6%**
- **Higher** improvement levels achieved under **higher** penalties
- Sequential approach is only subject to penalties
  - Objective decreases faster

Construction vs. Improvement



- Both phases are useful
- Construction solutions embed different potential for improvement

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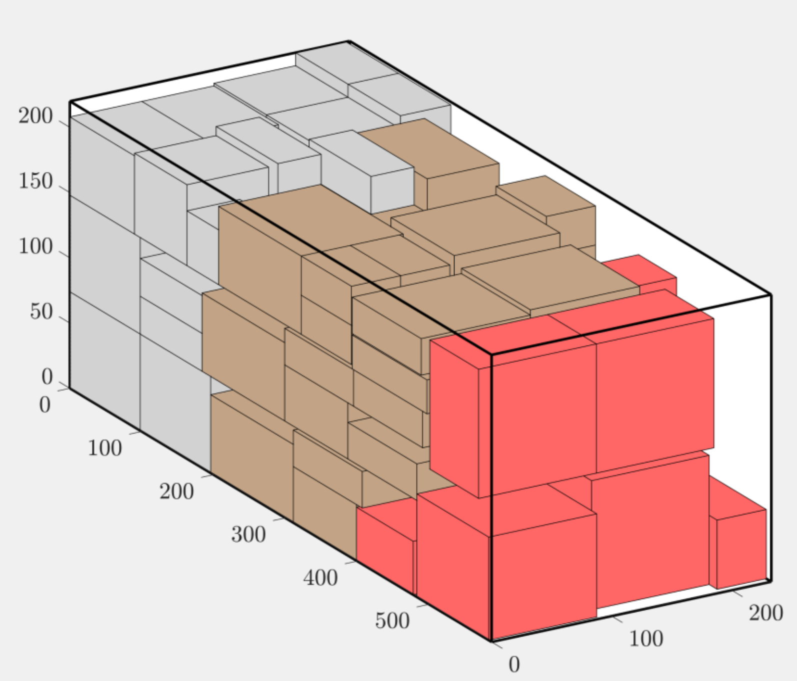
Comparison

- 3-customer example (BR7, instance 1)

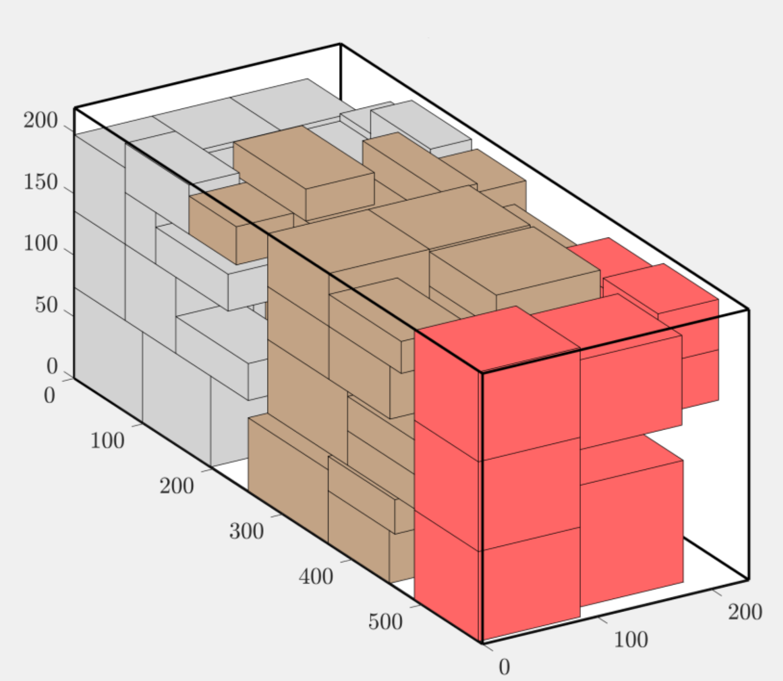
Construction



Objective: 74.6

Loaded items: 90

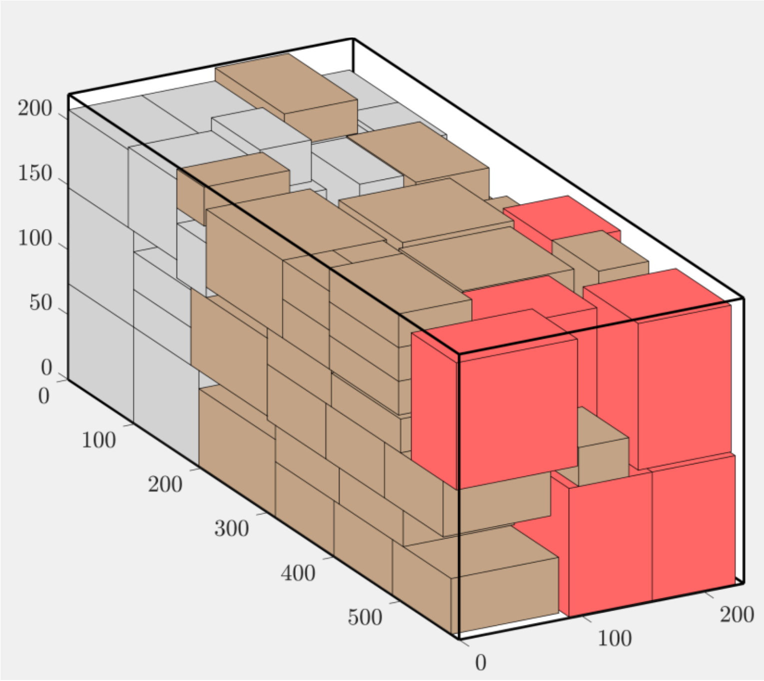
Construction (Hard)



Objective: 72.5

Loaded items: 88

Improved (Soft)



Objective: 81.9

Loaded items: 92

Trade-off cargo value vs. penalties

	Difference Improvement minus Construction											
	P1			P2			P3			P4		
	cargo	penalty	objective	cargo	penalty	objective	cargo	penalty	objective	cargo	penalty	objective
BR1	1.3	-0.4	1.7	1.2	-1.4	2.7	0.3	-3.8	4.1	0.1	-6.0	6.1
BR2	1.1	-0.2	1.3	0.7	-1.3	2.0	-0.4	-3.4	3.0	-1.6	-5.9	4.3
BR3	1.4	0.1	1.3	0.9	-1.4	2.3	0.3	-3.1	3.4	-1.4	-6.5	5.2
BR4	1.9	0.3	1.6	1.6	-0.4	2.0	0.6	-2.2	2.8	-1.1	-5.4	4.3
BR5	1.4	0.0	1.4	1.7	-0.9	2.6	-0.1	-4.0	4.0	-2.1	-8.0	5.9
BR6	2.1	0.4	1.7	2.2	-0.2	2.4	0.6	-2.9	3.5	-1.2	-6.2	5.1
BR7	2.2	0.4	1.8	1.7	-0.2	1.9	0.5	-2.7	3.3	-1.3	-5.4	4.1
BR8	2.4	0.4	2.0	2.5	0.2	2.4	1.3	-1.6	2.9	-0.4	-4.0	3.6
BR9	2.8	0.9	1.9	2.5	0.6	1.9	1.3	-1.3	2.6	-0.5	-3.9	3.4
BR10	2.7	0.6	2.1	2.5	0.6	1.9	1.0	-1.2	2.2	-0.6	-3.5	2.9
BR11	2.2	0.3	1.9	2.8	0.9	1.9	1.2	-0.7	2.0	-0.5	-3.0	2.5
BR12	2.6	0.6	2.0	2.6	0.9	1.7	1.2	-0.7	1.9	-0.7	-3.3	2.6
BR13	3.1	1.2	1.9	2.0	0.7	1.4	0.9	-0.8	1.7	-0.7	-2.8	2.1
BR14	2.6	0.7	1.8	2.3	0.8	1.6	1.0	-1.0	1.9	-1.0	-3.3	2.3
BR15	2.8	0.7	2.1	2.2	0.6	1.7	0.9	-1.1	2.0	-1.2	-3.5	2.3
Mean	2.1	0.4	1.7	2.0	0.0	2.1	0.7	-2.0	2.8	-1.0	-4.7	3.8

Region reconstruction method: Objectives

	P-1			P-2			P-3			P-4		
Class	M-1	M-2	M-3	M-1	M-2	M-3	M-1	M-2	M-3	M-1	M-2	M-3
BR1	83.7	83.6	**83.8**	79.8	79.3	**79.9**	**78.2**	76.2	78.1	**77.6**	74.4	**77.6**
BR2	**83.2**	**83.2**	**83.2**	**79.7**	79.2	78.9	**77.5**	76.1	77.2	**75.9**	73.4	75.3
BR3	80.9	**81.0**	80.9	**76.1**	75.6	75.7	**73.0**	71.4	72.4	**71.0**	68.0	70.2
BR4	80.3	**80.4**	80.2	**75.4**	75.1	75.0	**72.6**	71.6	72.5	**70.9**	68.3	70.4
BR5	79.6	**79.7**	79.6	**74.6**	73.7	73.6	**71.7**	69.8	70.8	**69.7**	66.4	68.8
BR6	**79.0**	78.8	78.7	**74.2**	73.5	73.6	**71.4**	69.8	70.5	**69.3**	67.3	68.4
BR7	**78.1**	78.0	77.7	**73.0**	72.7	72.4	**70.4**	69.0	69.4	**67.6**	65.5	66.9
BR8	**77.7**	77.5	77.1	**73.1**	72.5	71.9	**70.2**	69.3	69.0	**67.8**	66.2	66.7
BR9	**77.0**	76.9	76.4	**72.5**	72.2	71.6	**69.9**	68.9	68.8	**67.6**	66.1	66.3
BR10	**76.5**	76.2	75.7	**72.1**	71.7	71.0	**69.1**	68.4	68.2	**66.7**	65.3	65.7
BR11	**76.4**	76.3	75.8	**72.1**	71.6	71.3	**69.1**	68.5	68.3	**66.7**	65.8	65.7
BR12	**76.1**	75.9	75.5	**71.3**	70.9	70.4	**68.1**	67.6	67.2	**65.5**	64.5	64.6
BR13	**76.3**	76.1	75.5	**71.8**	71.7	71.1	**69.0**	68.4	68.1	**66.4**	65.4	65.6
BR14	**75.8**	75.7	75.2	**71.2**	70.8	70.2	**68.0**	67.3	66.8	**65.1**	64.4	64.0
BR15	**75.9**	75.5	74.8	**71.2**	70.8	70.1	**68.0**	67.3	66.8	**65.1**	64.2	64.5
Mean	**78.4**	78.3	78.0	**73.9**	73.4	73.1	**71.1**	70.0	70.3	**68.9**	67.0	68.0

- Recall that:
  - **M-1**: Conflicts
  - **M-2**: Penalty densities
  - **M-3**: Penalty densities & empty spaces
- Except for a few cases, M-1 performs the best on average

Region reconstruction method: Wins

	P-1			P-2			P-3			P-4		
Class	M-1	M-2	M-3	M-1	M-2	M-3	M-1	M-2	M-3	M-1	M-2	M-3
BR1	30	**46**	39	39	49	**52**	49	**58**	56	53	57	**58**
BR2	**35**	34	34	41	**50**	29	**47**	**47**	41	**61**	44	51
BR3	24	**46**	34	41	**43**	35	**54**	44	36	**61**	38	41
BR4	32	**40**	34	40	**41**	33	**45**	32	39	**56**	30	36
BR5	27	**42**	31	**53**	28	25	**49**	34	31	**56**	37	35
BR6	**45**	40	23	**51**	29	28	**59**	28	31	**56**	28	32
BR7	**44**	36	25	40	**44**	26	**53**	30	25	**55**	27	36
BR8	**48**	31	21	**61**	33	20	**61**	32	24	**59**	31	33
BR9	**46**	34	22	**57**	38	21	**69**	29	19	**66**	31	26
BR10	**47**	41	12	**63**	38	11	**62**	30	24	**66**	25	29
BR11	**48**	38	16	**61**	27	22	**52**	30	26	**60**	33	25
BR12	**48**	40	14	**56**	36	22	**57**	44	21	**63**	26	28
BR13	**43**	39	18	**44**	42	24	**58**	38	20	**61**	30	35
BR14	**42**	**42**	20	**54**	38	17	**60**	28	22	**60**	35	19
BR15	**53**	37	13	**55**	35	18	**65**	34	19	**58**	29	33
Mean	**40.8**	39.1	23.7	**50.4**	38.1	25.5	**56.0**	35.9	28.9	**59.4**	33.4	34.5

- More detailed picture at instance level
- M-1 rarely exceeded 55-60% of wins
  - The other methods can still be useful
  - Combine / randomize them?

Conclusion

- We studied a variant of the MDCLP with **soft unloading constraints**, which contrast the hard unloading constraints commonly found in the CLP and VRP literature

- We used **penalty functions** to model the indirect cost/time of relocating items during delivery
  - Activated when above, visibility, and reachability constraints are violated between pairs of boxes belonging to different customers

- We proposed:
  - an **MILP** to tackle small instances to optimality under specific penalties
  - a more general **heuristic framework** made of construction and improvement phases

- Our study shows that incorporating soft unloading constraints can be significantly **more efficient** that both sequential strategy (15%) and hard unloading constraints (12%)

Further work

- **Algorithmic enhancements**
  - Add movements to improvement phase (e.g., swapping items; see VNS by Parreño et al. 2010)
  - Try alternative merit functions during reconstruction
  - Use graph representation (Fekete et al. 2007) not only to shift boxes but to modify relative position of boxes using transitive orientations (Trivella and Pisinger 2016)

- Extend to **dynamic MDCLP** (more complex)
  - Unloading constraints are computed based on a static packing configuration
  - The multi-drop process allows repositioning cargo at different locations
  - Accounting for extra decisions to where to relocate items adds flexibility

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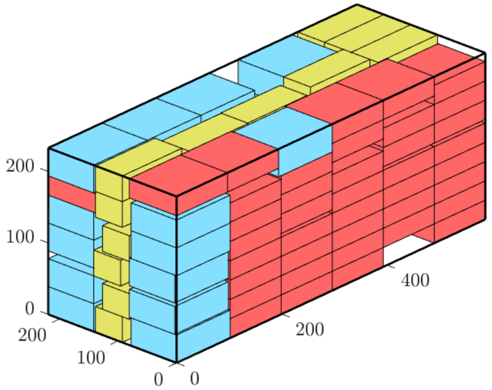

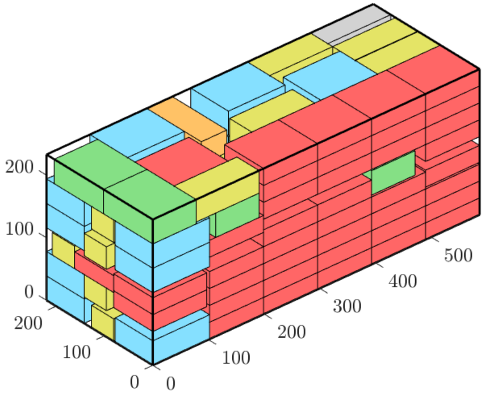

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Paper available online → ssrn.com/abstract=3929994

Full MILP formulation

$$
\begin{aligned}
\max \quad & \sum_{i\in\mathcal{B}} \pi_i\, t_i - \sum_{i\in\mathcal{B}} \sum_{j\in\mathcal{B}: c_i<c_j} \left[p_{ij}\left(\alpha_{\mathrm{A}} q_j + \beta_{\mathrm{A}} v_j + \gamma_{\mathrm{A}}\right) + r_{ij}\left(\alpha_{\mathrm{V}} q_j + \beta_{\mathrm{V}} v_j + \gamma_{\mathrm{V}}\right)\right] & \\
\text{s.t.}: \quad & f_{ij} + f_{ji} + b_{ij} + b_{ji} + u_{ij} + u_{ji} + (1-t_i) + (1-t_j) \geq 1 & \forall i,j\in\mathcal{B},\ i<j, \\
& x_i + l_i(o_{i1}+o_{i2}) + w_i(o_{i3}+o_{i4}) + h_i(o_{i5}+o_{i6}) - x_j \leq L(1-b_{ij}) & \forall i,j\in\mathcal{B}, \\
& y_i + l_i(o_{i3}+o_{i5}) + w_i(o_{i1}+o_{i6}) + h_i(o_{i2}+o_{i4}) - y_j \leq W(1-f_{ij}) & \forall i,j\in\mathcal{B}, \\
& z_i + l_i(o_{i4}+o_{i6}) + w_i(o_{i2}+o_{i5}) + h_i(o_{i1}+o_{i3}) - z_j \leq H(1-u_{ij}) & \forall i,j\in\mathcal{B}, \\
& x_i + l_i(o_{i1}+o_{i2}) + w_i(o_{i3}+o_{i4}) + h_i(o_{i5}+o_{i6}) \leq L & \forall i,j\in\mathcal{B}, \\
& y_i + l_i(o_{i3}+o_{i5}) + w_i(o_{i1}+o_{i6}) + h_i(o_{i2}+o_{i4}) \leq W & \forall i,j\in\mathcal{B}, \\
& z_i + l_i(o_{i4}+o_{i6}) + w_i(o_{i2}+o_{i5}) + h_i(o_{i1}+o_{i3}) \leq H & \forall i,j\in\mathcal{B}, \\
& o_{i1} + o_{i2} + o_{i3} + o_{i4} + o_{i5} + o_{i6} = 1 & \forall i,j\in\mathcal{B}, \\
& x_j \leq x_i + l_i(o_{i1}+o_{i2}) + w_i(o_{i3}+o_{i4}) + h_i(o_{i5}+o_{i6}) + L\, b_{ij} & \forall i,j\in\mathcal{B},\ c_i\neq c_j, \\
& y_j \leq y_i + l_i(o_{i3}+o_{i5}) + w_i(o_{i1}+o_{i6}) + h_i(o_{i2}+o_{i4}) + W\, f_{ij} & \forall i,j\in\mathcal{B},\ c_i\neq c_j, \\
& z_j \leq z_i + l_i(o_{i4}+o_{i6}) + w_i(o_{i2}+o_{i5}) + h_i(o_{i1}+o_{i3}) + H\, u_{ij} & \forall i,j\in\mathcal{B},\ c_i\neq c_j, \\
& b_{ij} + b_{ji} + f_{ij} + f_{ji} \geq 1 - a_{ij} & \forall i,j\in\mathcal{B},\ c_i<c_j, \\
& b_{ij} + b_{ji} + f_{ij} + f_{ji} \leq 2(1 - a_{ij}) & \forall i,j\in\mathcal{B},\ c_i<c_j, \\
& f_{ij} + f_{ji} + u_{ij} + u_{ji} \geq 1 - d_{ij} & \forall i,j\in\mathcal{B},\ c_i<c_j, \\
& f_{ij} + f_{ji} + u_{ij} + u_{ji} \leq 2(1 - d_{ij}) & \forall i,j\in\mathcal{B},\ c_i<c_j, \\
& p_{ij} + 1 \geq a_{ij} + u_{ij} & \forall i,j\in\mathcal{B},\ c_i<c_j, \\
& r_{ij} + 1 \geq d_{ij} + b_{ij} & \forall i,j\in\mathcal{B},\ c_i<c_j, \\
\text{var.}: \quad & b_{ij},\ f_{ij},\ u_{ij},\ a_{ij},\ d_{ij},\ p_{ij},\ r_{ij} \in \{0,1\} & \forall i,j\in\mathcal{B}, \\
& t_i,\ o_{i1},\ o_{i2},\ o_{i3},\ o_{i4},\ o_{i5},\ o_{i6} \in \{0,1\} & \forall i\in\mathcal{B}, \\
& x_i,\ y_i,\ z_i \geq 0 & \forall i\in\mathcal{B}.
\end{aligned}
$$

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