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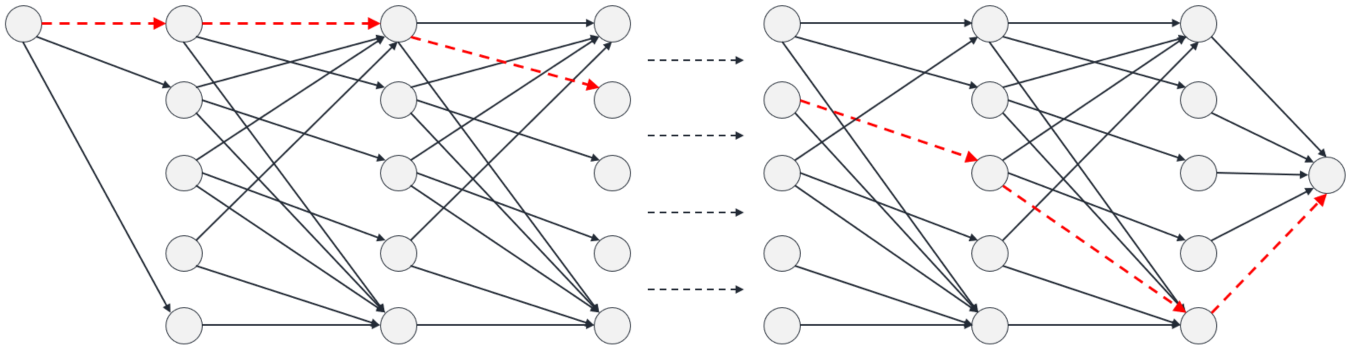


EWG TRANSPORTATION

The Multi-Commodity Network Flow Problem with Soft Transit Time Constraints: Application to Liner Shipping

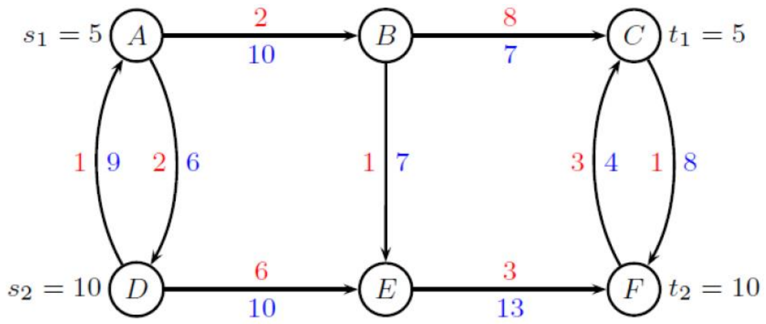
Alessio Trivella

Joint work with: Francesco Corman, David F. Koza, David Pisinger

Standard multi-commodity network flow

Multi-commodity network flow problem (**MCNF**):

route a set of commodities through a capacitated network, from their respective origins to demand destinations, minimizing transportation cost while respecting capacity



- Widely applied, e.g., in transportation and telecommunication problems (Ahuja et al. 1993)
- In **liner shipping**, used to optimally route containers in maritime shipping networks
- Does not consider commodity transit times

Hard transit time constraints

- In liner shipping, commodity transit time is a **critical factor** to ensure competitive service levels (Notteboom 2006, Gelareh et al. 2010, Brouer et al. 2013, Meng et al. 2013, Karsten et al. 2015)

- Researchers have considered the *hard time-constrained MCNF* (**HTC-MCNF**) (Holmberg and Yuan 2003, Wang and Meng 2014, Karsten et al. 2015, Koza et al. 2020)

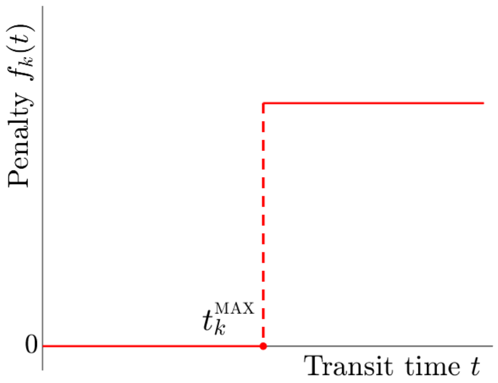
  → Commodities subject a maximum allowed transit time

- Issues with HTC-MCNF (after discussions with a major liner operator):
  1. Solutions exceeding the target transit time **are discarded**, even when the delay is tiny
  2. Paths below the target transit time **are equivalent**, independent of the transit time

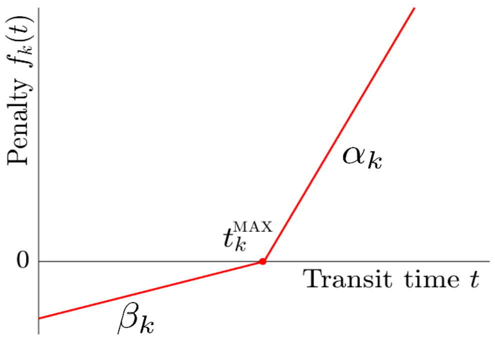
Soft transit time constraints

- To overcome these issues, we introduce the *soft time-constrained MCNF* (**STC-MCNF**)
  - → We do not exclude a priori longer routes but punish them using a penalty
  - → We encourage the use of faster routes through a discount

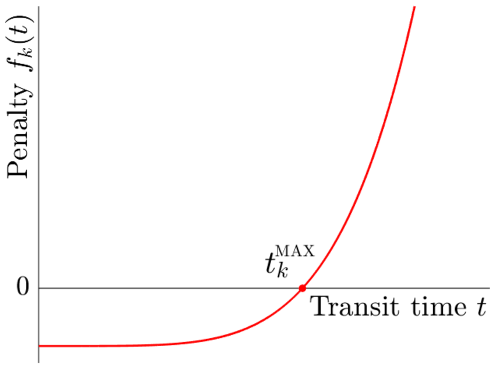
Piecewise constant (Brouer et al. 2013)



Piecewise linear (Reinhardt et al. 2020)



Convex (Pisinger 2016)

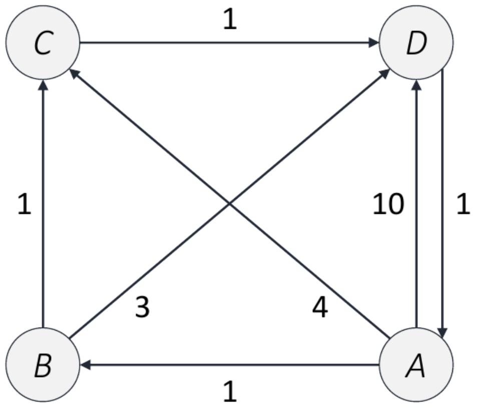


- Compared to the standard MCNF, arcs also have a transit time duration, and the objective is to **minimize the sum of arc traversal costs and delay penalties**

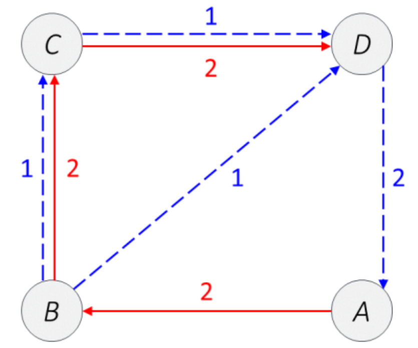
Illustrative example

Consider this small instance with only two commodities (all arcs: capacity 2, transit time 1)

k	o_k	s_k	d_k	t_k^{MAX}	α_k	β_k
1	A	D	2	1	4	0
2	B	A	2	2	3	0



Solutions:

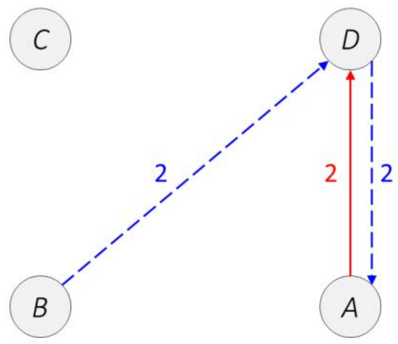


~~MCNF~~

~~Cost: 13~~

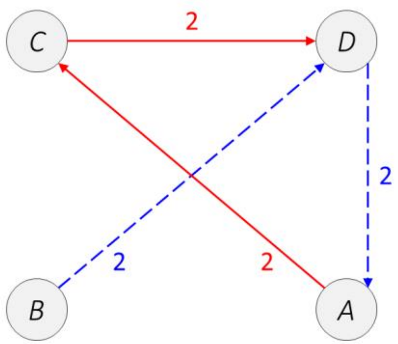
SH

Cost: **32**



HTC-MCNF

Cost: **28**



STC-MCNF

Cost: **26**

Properties

COST DIVERGENCE

Sequences of instances exist for which (keeping the same network but changing some parameters):

1. the cost difference between HTC-MCNF and STC-MCNF diverge
2. the cost difference between SH and STC-MCNF diverge

ARC COST MODIFICATION

Instances exist such that:

solving STC-MCNF is not equivalent to solving MCNF in a network with arbitrarily modified arc cost

COMPLEXITY

The STC-MCNF is:

- NP-hard, in general
- Weakly NP-hard, if arc transit times are integer values

Column generation

The path formulation for STC-MCNF is a standard set covering-like model

$$\min \sum_{k\in\mathcal{K}} \sum_{p\in\mathcal{P}^k} c'_p\, x^k_p$$

$$\text{s.t.:} \sum_{p\in\mathcal{P}^k} x^k_p = d_k, \qquad \forall k \in \mathcal{K},$$

$$\sum_{k\in\mathcal{K}} \sum_{p\in\mathcal{P}^k_{(i,j)}} x^k_p \leq u_{ij}, \qquad \forall (i,j) \in \mathcal{A},$$

$$x^k_p \geq 0, \qquad \forall k \in \mathcal{K},\, p \in \mathcal{P}^k.$$

where $c'_p := c_p + f_k(t_p)$

- Master: restricted version of this model
- Pricing: variant of the resource-constrained shortest path problem (RCSP)
- We solve it with a dynamic programming algorithm (Irnich and Desaulniers 2005).
- Add paths to the master if the "penalized reduced cost" is negative:

$$\bar{c}_p = \sum_{(i,j)\in p} (c_{ij} - \gamma_{ij}) - \eta_k + f_k\left(\sum_{(i,j)\in p} t_{ij}\right)$$

RCSP strategies

- Choose one-to-all vs. single source RCSP + adapt dominance rules to STC:

◇ $\mathfrak{D}1$ [Standard]. *Label* (C_i^h, T_i^h) *is* $\mathfrak{D}1$*-dominated by another label* (C_i^l, T_i^l) *for the same node if* $C_i^l \leq C_i^h$ *and* $T_i^l \leq T_i^h$

◇ $\mathfrak{D}3$ [Time]. *Label* (C_i^h, T_i^h) *is* $\mathfrak{D}3$*-dominated if* $T_i^h + t_{i \to s_k}^{\text{MIN}} > t_k^{\text{MAX}}$

◇ $\mathfrak{D}4$ [Cost]. *Label* (C_i^h, T_i^h) *is* $\mathfrak{D}4$*-dominated if* $C_i^h + \bar{c}_{i \to s_k}^{\text{MIN}} > \eta_k$

OK for HTC-MCNF

For STC-MCNF we use an upper bound on transit time

Relaxations in case of one-to-all RCSP:

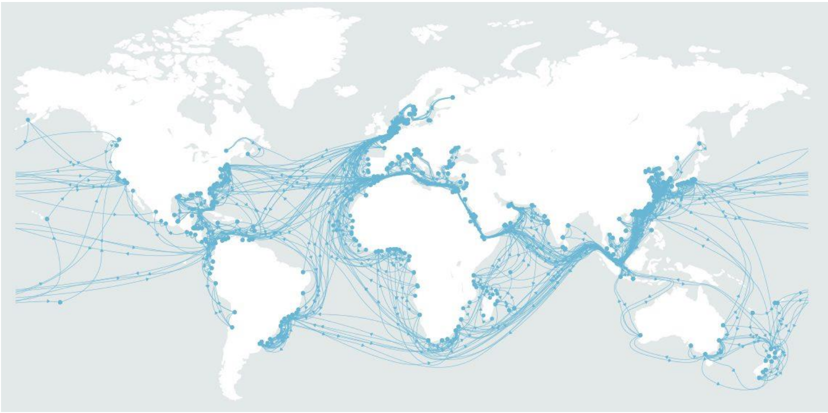
$$T_i^h + \min\{t_{i \to s_k}^{\text{MIN}},\ k \in \mathcal{K}_j\} > \max\{t_k^{\text{MAX}},\ k \in \mathcal{K}_j\}$$
$$C_i^h + \min\{\bar{c}_{i \to s_k}^{\text{MIN}},\ k \in \mathcal{K}_j\} > \max\{\eta_k,\ k \in \mathcal{K}_j\}$$

◇ $\mathfrak{D}5$ [Goal]. *Label* (C_i^h, T_i^h) *is* $\mathfrak{D}5$*-dominated if there exists a label* $l = (C_{s_k}^l, T_{s_k}^l) \in \mathcal{L}_{s_k}$ *such that* $C_i^h + C_i^{\text{MIN}} + f(T_i^h + T_i^{\text{MIN}}) \geq C_{s_k}^l + f(T_{s_k}^l)$.

Case study

- Our experiments are based on the LINER-LIB benchmark instance (Brouer et al. 2013)
- We use scheduled networks from Koza et al. (2020)
- 5 instance classes $\{\texttt{WAF}, \texttt{MED}, \texttt{PAC}, \texttt{WS}, \texttt{EUA}\}$
- 12 instances per class (all results will be averages over these 12 instances)

Class	Description	$\lvert\mathcal{N}\rvert$	$\lvert\mathcal{N}^O\rvert$	$\lvert\mathcal{A}\rvert$	$\lvert\mathcal{K}\rvert$
WAF	West Africa	69.5	16	270.9	30.1
MED	Mediterranean	101.6	26.1	498.1	247.3
PAC	Pacific region	189.7	34.2	1567.7	581.7
WS	World small	356.1	44.8	4312.5	1615.9
EUA	Europe Asia	353.7	67.0	4313.7	2428.8



Source: Maersk Line

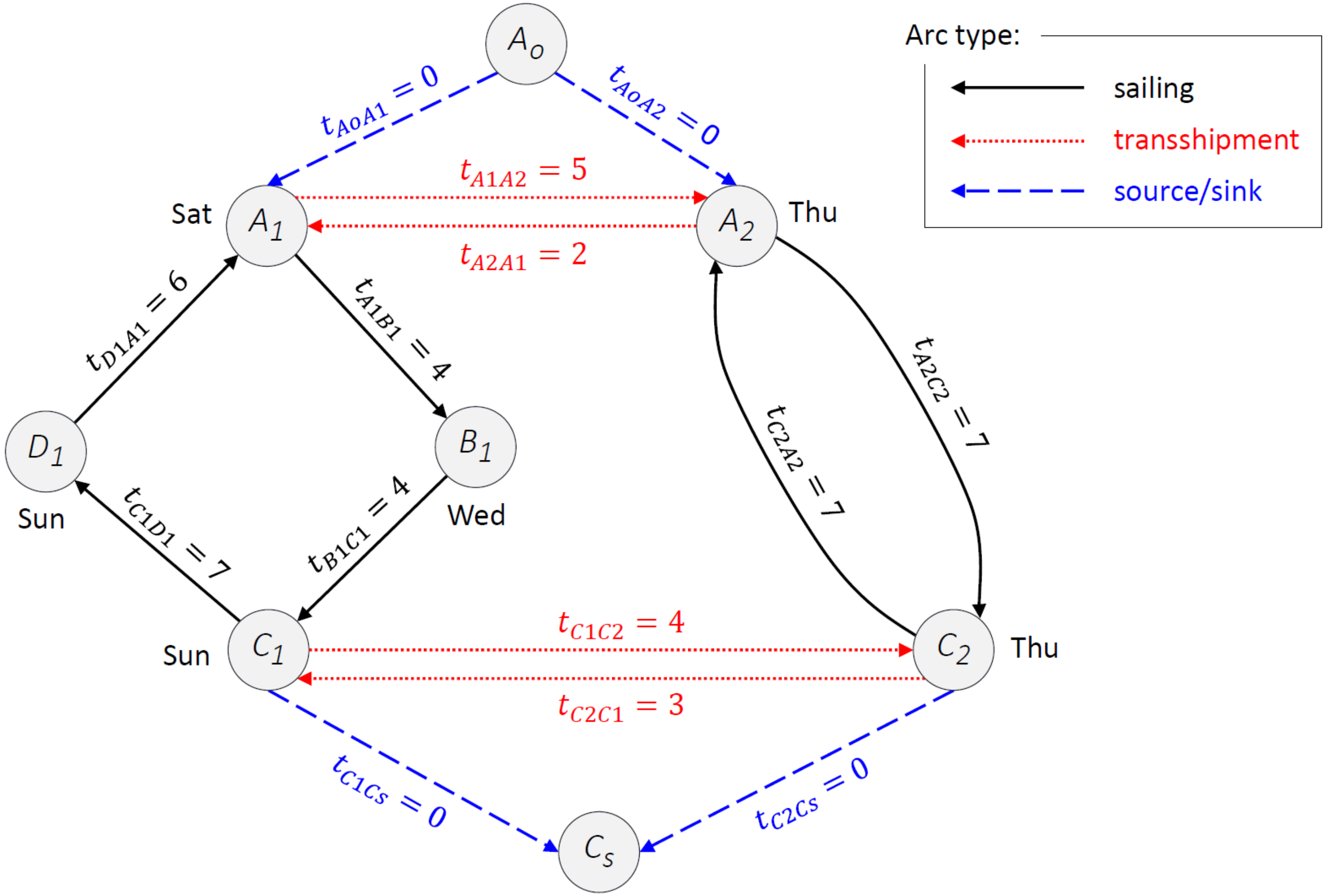
Network construction

Nodes:

- Port calls
- Source
- Sink

Arcs:

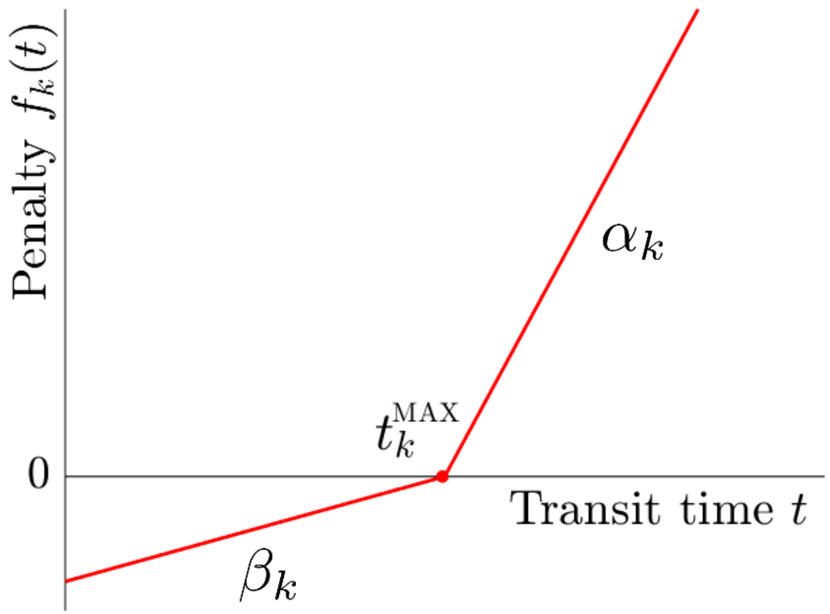
- Sailing
- Transshipment
- Source
- Sink



Penalty choice

We use the penalties from Reinhardt et al. (2020)

- Two slopes: **delay cost** and **early arrival bonus**
- Penalties and parameters were validated by a major liner shipping operator
- We tested **variations** of this penalty where the slopes and the transit time limits are modified



Running time (1/2)

Column generation details

	MCNF with CG					SH	STC-MCNF with CG				
			Running time (s)			Running time (s)			Running time (s)		
Class	Iter.	Paths	Master	Pricing	Total		Iter.	Paths	Master	Pricing	Total
WAF	6.9	67.2	0.06	< 0.01	0.07	0.07	5.8	52.7	0.05	0.01	0.07
MED	4.9	445.5	0.06	0.01	0.07	0.07	4.4	352.4	0.05	0.04	0.09
PAC	9.9	2071.7	0.25	0.12	0.38	0.38	7.5	1185.6	0.11	0.34	0.46
WS	15.0	9593.2	3.15	2.00	5.18	5.18	10.4	4523.6	0.65	3.25	3.83
EUA	12.8	12499.8	3.03	2.19	5.24	5.24	8.9	5788.4	0.65	3.29	3.96

Single-source shortest path

Class	Baseline	$\mathfrak{D}3$	$\mathfrak{D}4$	$\mathfrak{D}5$	$\mathfrak{D}3+\mathfrak{D}4$	$\mathfrak{D}3+\mathfrak{D}5$	$\mathfrak{D}4+\mathfrak{D}5$	$\mathfrak{D}3+\mathfrak{D}4+\mathfrak{D}5$
WAF	0.016	0.009	0.012	0.012	0.006	0.009	0.012	0.006
MED	0.110	0.051	0.097	0.108	0.033	0.042	0.073	0.031
PAC	2.340	0.607	1.703	2.291	0.342	0.375	1.625	0.330
WS	41.97	5.406	21.62	39.20	3.147	3.456	18.86	2.910
EUA	29.23	5.168	18.38	27.45	3.397	3.909	16.31	3.245
Speedup	-	68.2%	29.9%	8.3%	79.5%	73.6%	37.7%	80.4%

One-to-all

Class	Baseline	$\mathfrak{D}3_R$	$\mathfrak{D}4_R$	$\mathfrak{D}3_R+\mathfrak{D}4_R$
WAF	0.011	0.005	0.006	0.004
MED	0.020	0.018	0.016	0.015
PAC	0.170	0.135	0.157	0.134
WS	1.404	1.012	1.389	1.067
EUA	0.954	0.768	0.927	0.760
Speedup	-	26.5%	15.4%	30.8%

Running time (2/2)

- Best implementation is one-to-all shortest path with the tightest set of dominance rules
- All instances can be solved quickly (**within 2 seconds**)
- Note that algorithm is optimized: multiple dominance rules, warm starting, multiple added columns etc.

- Path formulation/column generation is the way to go (we formulated an arc model with techniques to speed it up, but many instances were not solvable)
- Promising for incorporation into the **network design problem**

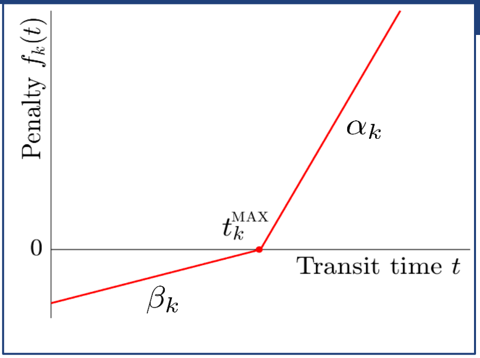
Objective function and cost decomposition

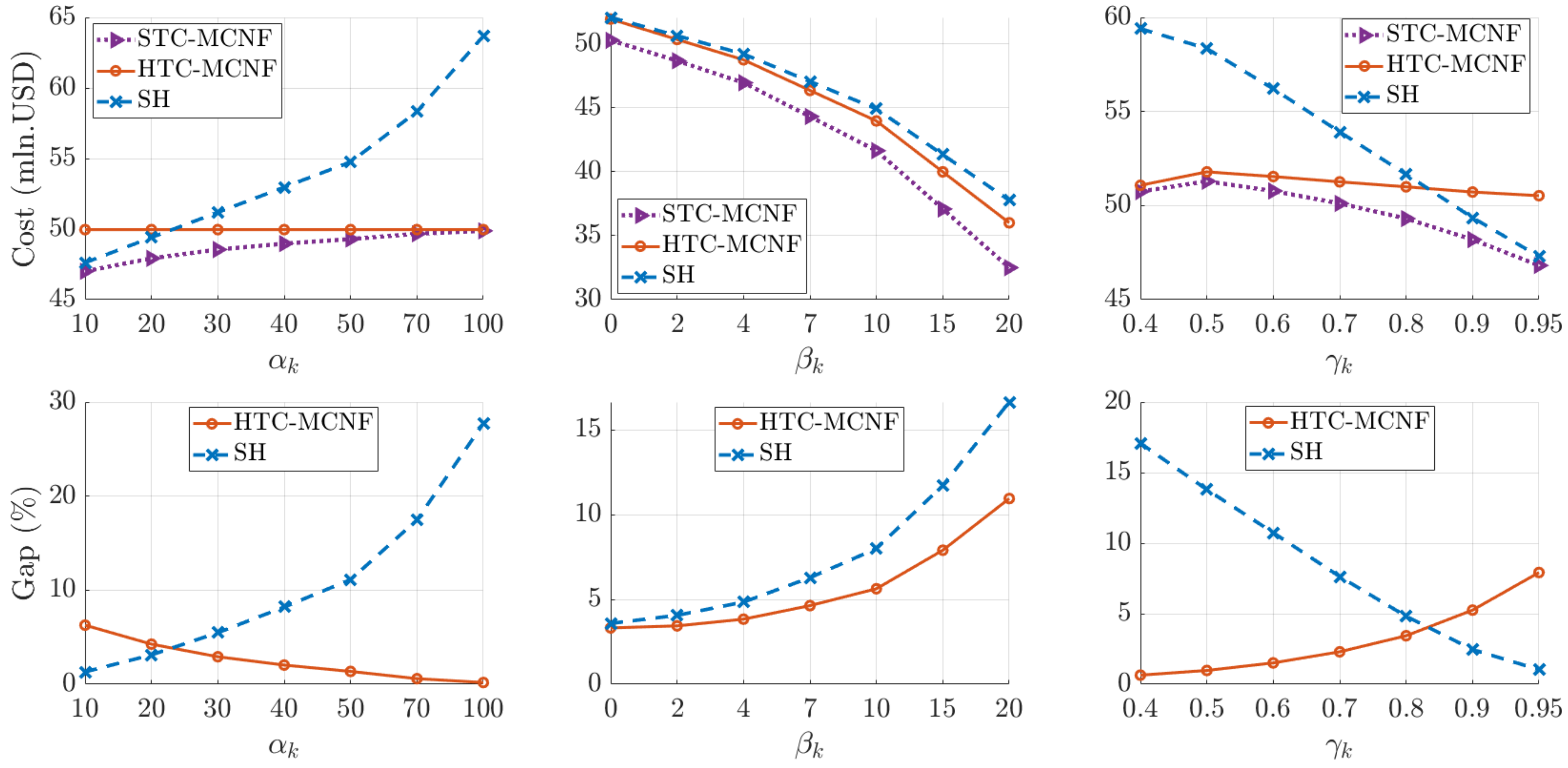
*Costs in mln.USD

	STC-MCNF			HTC-MCNF				SH			
Class	flow	pen.	total	flow	pen.	total	gap	flow	pen.	total	gap
WAF	3.79	-0.12	3.67	3.82	-0.13	3.69	0.49	3.78	-0.08	3.71	0.85
MED	2.48	0.06	2.53	2.56	-0.01	2.55	0.65	2.42	0.25	2.67	5.25
PAC	23.47	-0.92	22.54	23.64	-0.91	22.73	0.82	22.89	0.85	23.74	5.30
WS	83.26	0.35	83.61	87.14	-1.26	85.88	2.74	81.64	6.18	87.82	5.08
EUA	48.92	0.39	49.31	51.95	-0.96	51.00	3.44	47.62	4.03	51.66	4.82

- SH lowest flow cost but higher penalties → higher total cost than STC-MCNF
- HTC-MCNF highest flow cost → higher total cost than STC-MCNF
- Gap can be significant resulting in large cost difference

Cost and optimality gap at varying penalty (EUA)





Cost slope for delays

P1

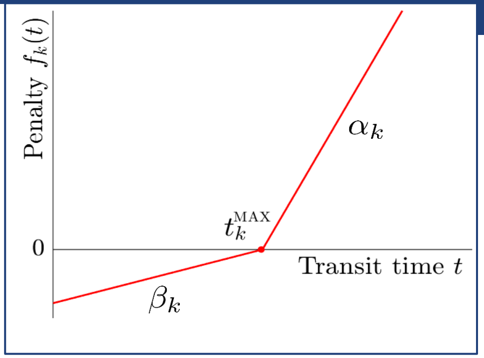
Bonus slope for early arrival

P2

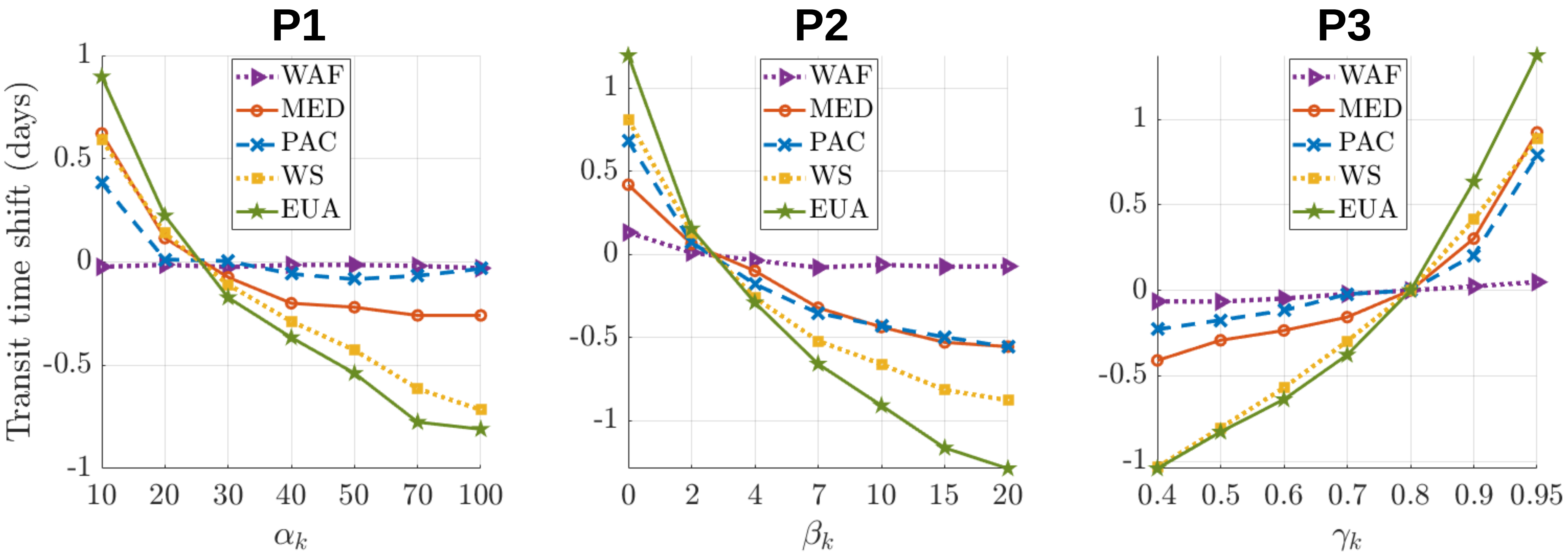
Hard limit shift

P3

Impact of penalties on transit time



- We consider a "flow-weighted" transit time $\frac{\sum_{k\in\mathcal{K}}\sum_{p\in\mathcal{P}_k^*} t_p\, x_p^k}{\sum_{k\in\mathcal{K}} d_k}$

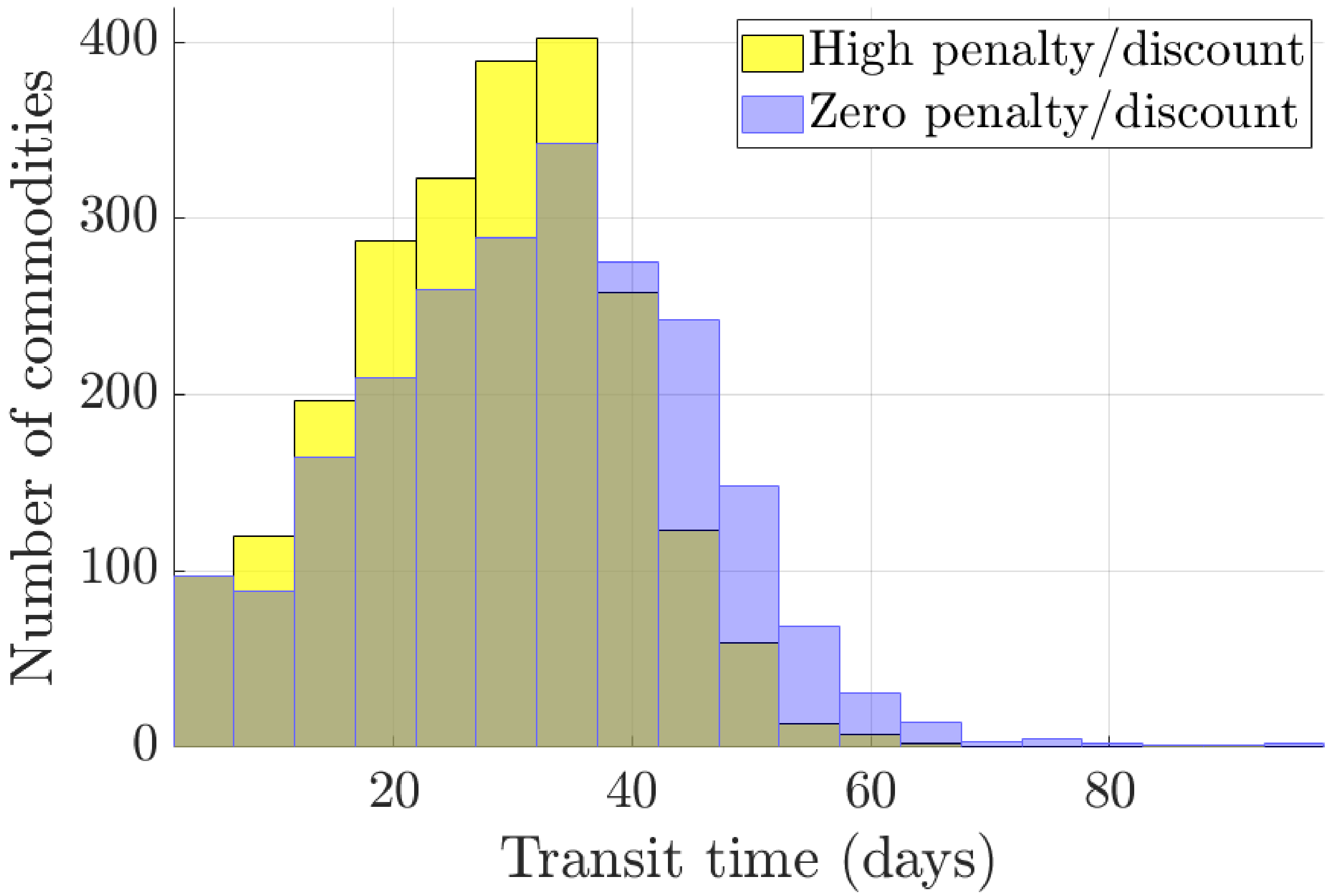


- Penalty is a lever to steer the flow towards faster or slower routing configurations

Transit time distribution (EUA)

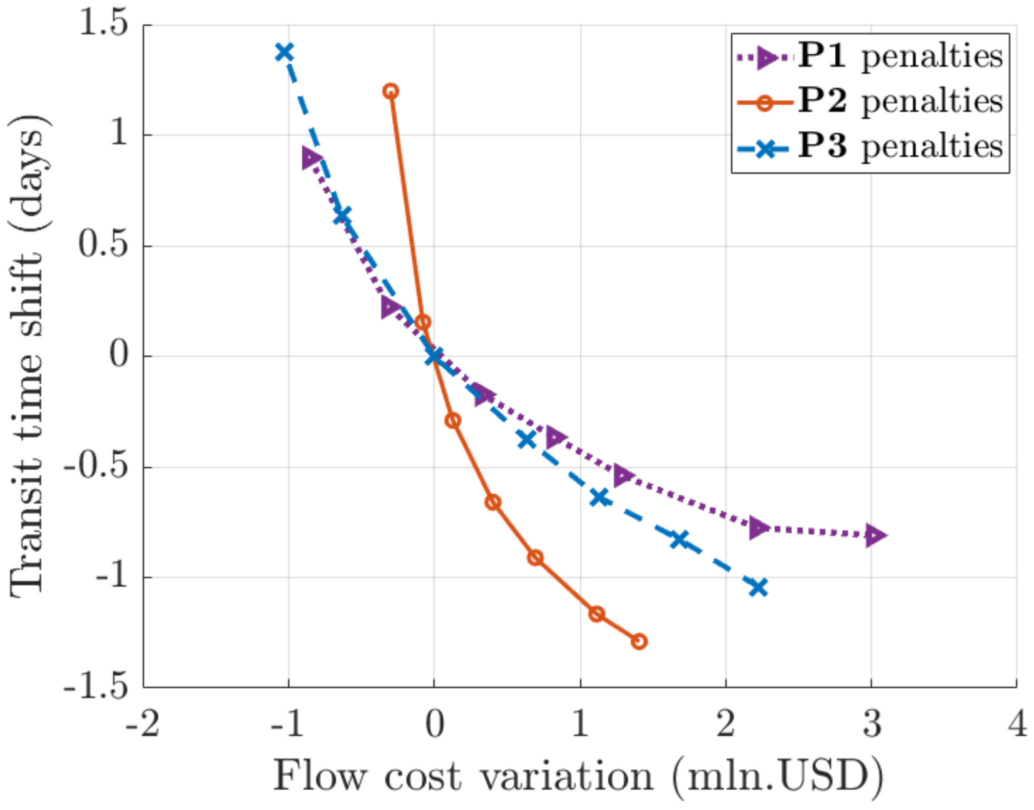
We use a high penalty and study the distribution of flow-weighted commodity transit time

- The transit time distribution with penalties is visibly shifted to the left: flow faster
- The distributions average differs by more than 4 days



Cost and transit time trade-off (EUA)

We consider the cost of the flow alone (i.e., the “real”/tangible cost)



- **P2** manages the “speed up” side of the trade-off more efficiently
- Transit time reduction of **1.3 days (3.6%)** achieved for a cost increase of **1.4 mln.USD (2.8%)**
- Speeding up flow is quite expensive, but liner operators may decide which flow to speed up

Conclusion

- We study a soft time-constrained version of the MCNF relevant in liner shipping
- We derive some properties of STC-MCNF and adapt a CG procedure to solve it
- On realistic liner-shipping instances (LINER-LIB), we examine solution cost, gap, and transit times, at varying penalty functions. Key takeaways:

1. Penalties can be used a **lever to steer the flow** towards slower/faster configurations
2. The operator must manage a **trade-off between flow cost and transit time**
3. The extra cost of reducing transit time should be balanced by (1) customer satisfaction increase and (2) customer churn decrease, with are hard-to-assess indirect benefits

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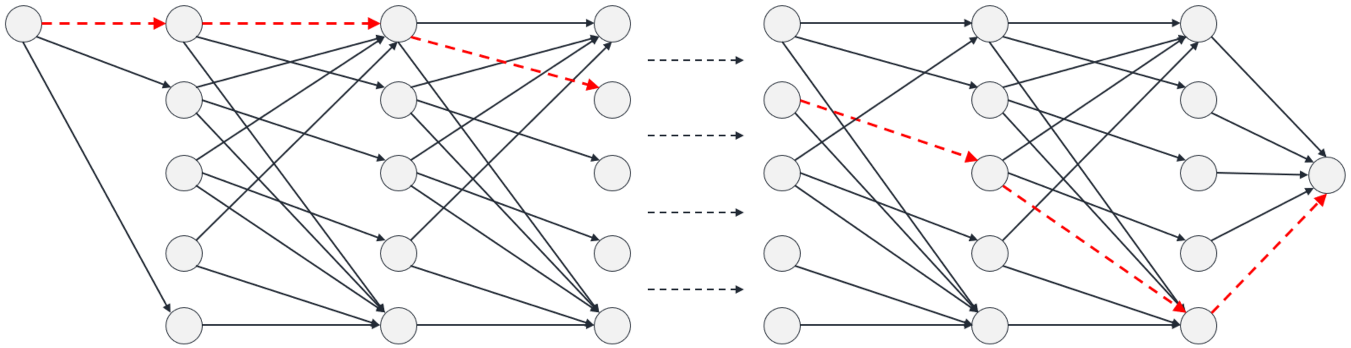

TRANSPORTATION

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Soft transit time constraints

- To overcome these issues, we introduce the *soft time-constrained MCNF* (**STC-MCNF**)

→ It does not exclude a priori longer routes but punish them using a penalty which is a function of the delay

→ It encourages the use of faster routes through a discount

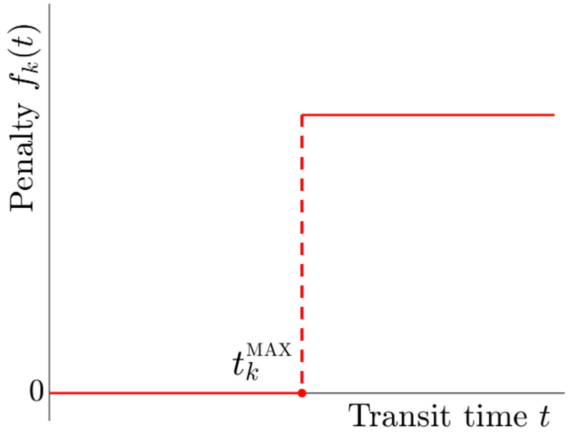
- Compared to the standard MCNF, arcs also have a transit time duration and the objective is to **minimize the sum of arc traversal costs and delay penalties**

- Not considered in the liner shipping literature for cargo routing. Used for optimizing the speed of vessels (Brauer et al. 2013, Reinhardt et al. 2020) or in the form of time-dependent commodity demand (Wang et al. 2013, Wang et al. 2016)

Penalty functions

- Functions that allow to model transit time in a more **flexible** manner
- Defined for each commodity and can be different (e.g., higher priorities, perishable goods)
- Example of penalties used in the liner shipping literature:

Piecewise constant (Brouer et al. 2013)



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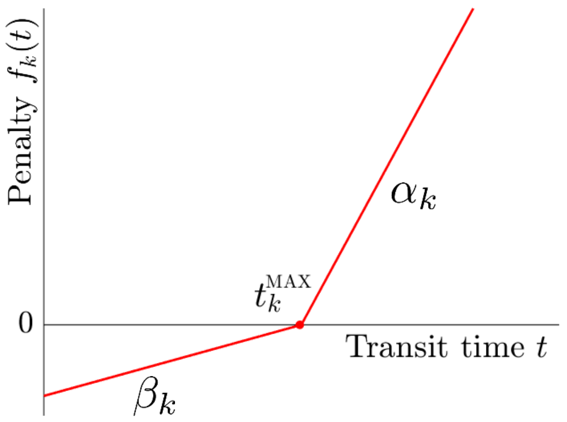

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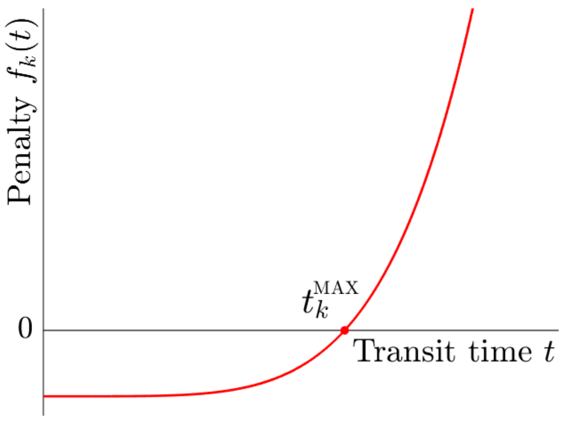

Standard multi-commodity network flow

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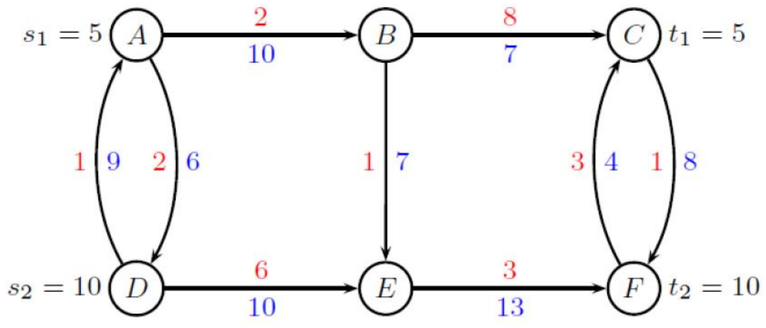

- Widely applied, e.g., in transportation and telecommunication problems (Ahuja et al. 1993)
- In **liner shipping**, used to optimally route containers in maritime shipping networks
- Can be tackled using:
  1. An "**arc formulation**", a linear program, still intractable for large problems
  2. A "**path formulation**", efficiently solvable with column generation
- Does not consider commodity transit times