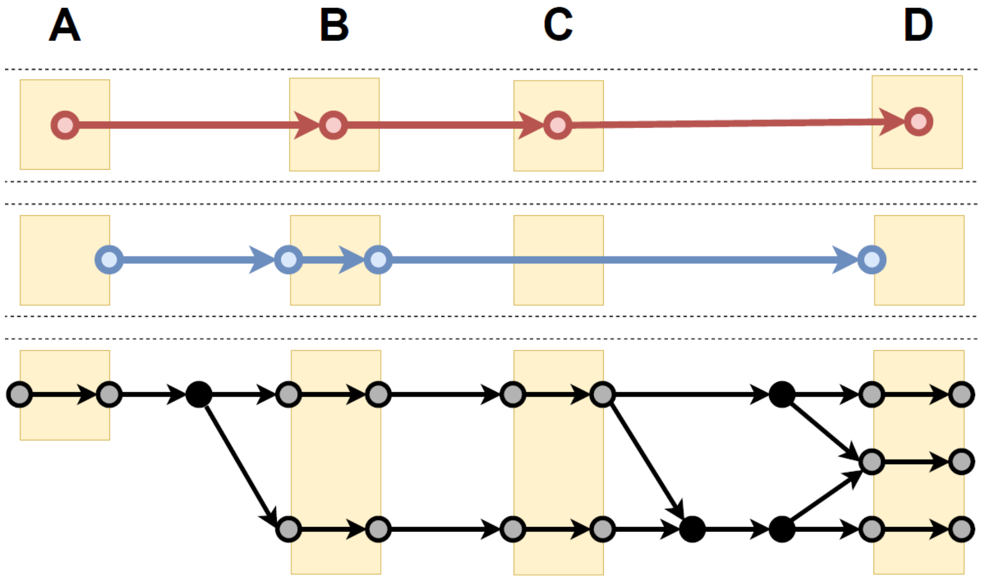

Enhancing the Interaction of Railway Timetabling and Line Planning with Infrastructure Awareness

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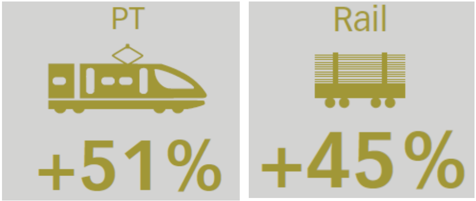
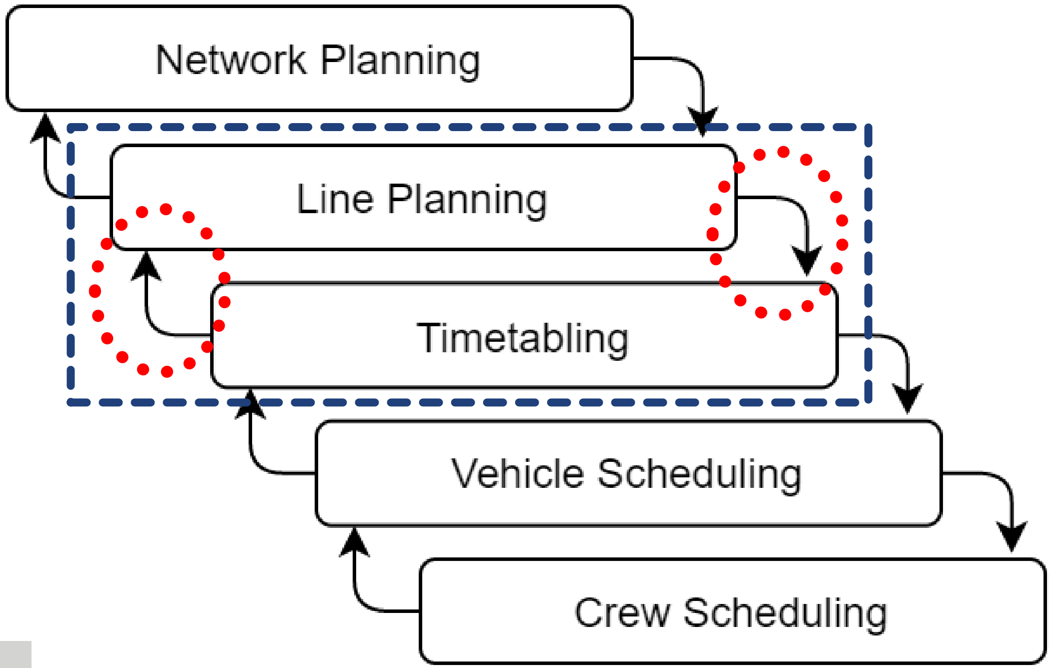
Planning a Railway System

- Quality and efficiency of a railway system largely depend on its planning and operation
- Planning such a system is complex as it is comprised of multiple stages
- We jointly study periodic line planning and timetabling with focus on their interaction
- **Relevance**: existing infrastructure needs to be used more efficiently to meet the expected increase in demand



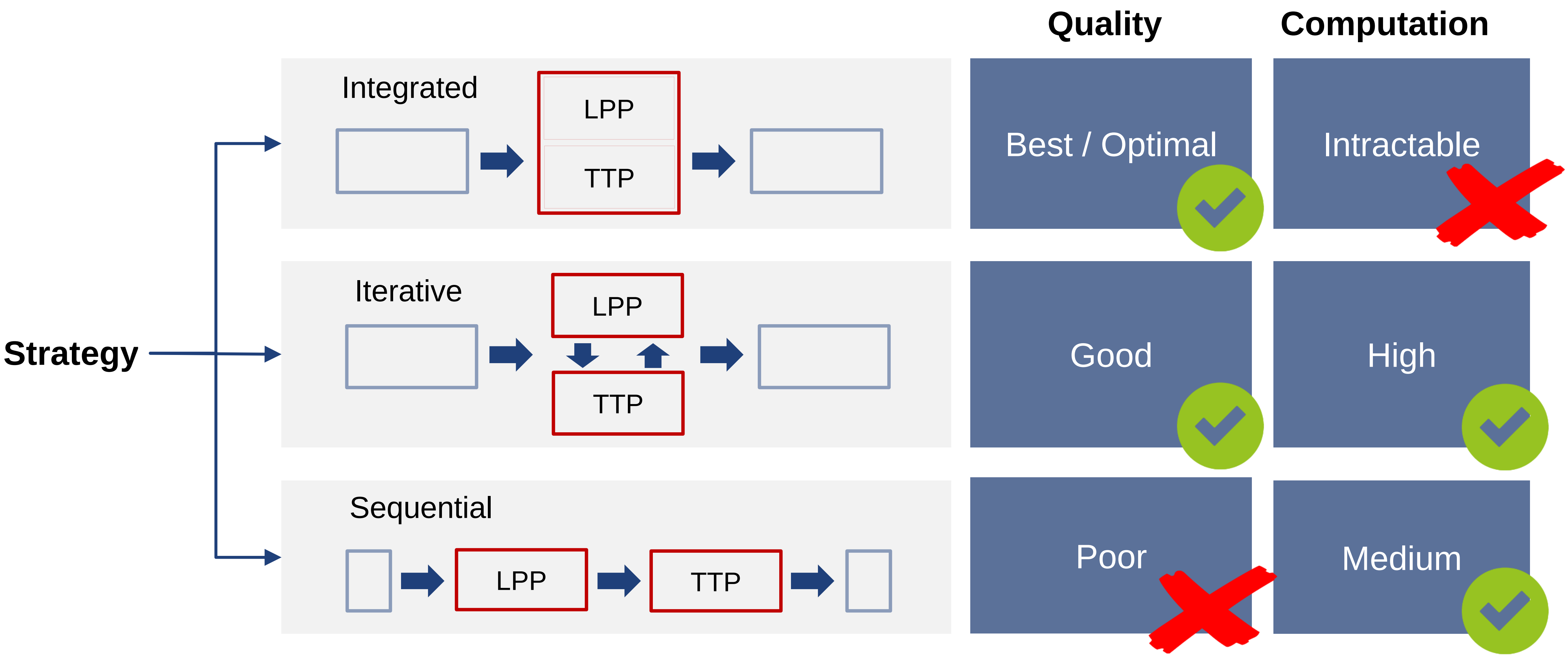
CH transport outlook 2040

Planning Stages



From Liebchen et al. (2004)

Line Planning (LPP) and Timetabling (TTP)



Related Literature

- The line planning problem is well studied (Schoebel and Scholl 2006, Goossens et al. 2006, Borndorfer et al. 2007, Schoebel 2012, Schmidt and Schoebel 2015, Goerigk and Schmidt 2017, Gattermann et al. 2017, Patzold et al. 2017)

- Periodic timetabling is also well studied (Liebchen and Moehring 2004, Grossmann 2011, Gattermann et al. 2016, Robenek et al. 2016, Grossmann 2016, Caimi et al. 2017, Wust et al. 2019, Herrigel et al. 2018, Borndoerfer et al. 2020)

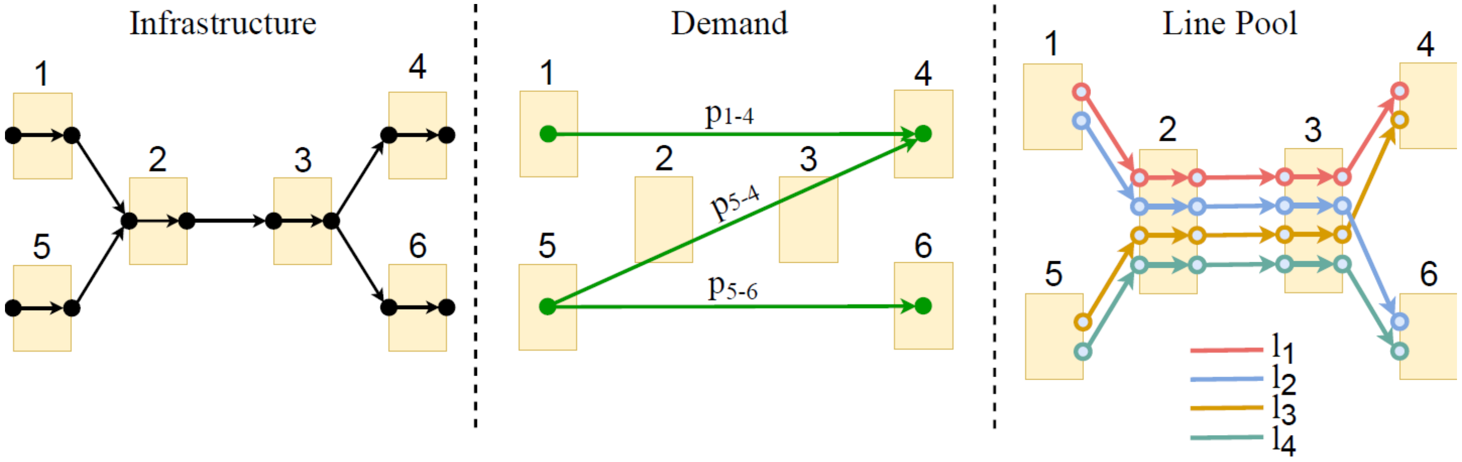
- Joint consideration of LPP and TTP scales poorly with integrated approaches (Lubbecke et al. 2018, Schiewe 2020). Iterative methods rely on simplified approaches, such as:
  - Discarding infeasible line plans (Burggraeve et al. 2017)
  - Restricting the frequencies of all lines (Yan and Goverde 2019)
  - Completely ignoring infrastructure (Fuchs and Corman 2019)

Summary of Contribution

1. We **theoretically** assess the benefit of banning conflicts vs. banning line plans
2. We provide a domain model and problem formulation enabling **train itinerary assignment** during timetabling, fully exploiting the available infrastructure
3. We propose a novel iterative approach that accurately identifies and bans the **smallest set of conflicting services** to find conflict-free solutions
4. We conducted a numerical study based on **real railway instances**, underscoring the value of precise conflict detection and of assigning itineraries while timetabling

Illustrative Example

- Based on example by Schiewe (2020)
- 6 stations, 4 lines
- Other parameters assigned



LPP valid solutions with objective score

	Configuration				Travel time			
Sol.	x_{l_1}	x_{l_2}	x_{l_3}	x_{l_4}	$t_{p_{1-4}}$	$t_{p_{5-4}}$	$t_{p_{5-6}}$	t_{total}
S-1	1	1	1	1	63	42	21	126
S-2	1	0	1	1	63	42	21	126
S-3	1	1	1	0	63	42	26	131
S-4	1	0	0	1	63	52	21	136
S-5	1	1	0	1	63	52	21	136
S-6	0	1	1	1	78	42	21	141
S-7	0	1	1	0	78	42	21	141

- Assume conflict in link 3 - 4
- Banning line plans: **3/4 iterations**
  (S-2 → S-3 → S-4/S-5)
  (S-1 → S-2 → S-3 → S-4/S-5)
- Banning conflict: **2 iterations**
  (S-1/S-2 → S-4/S-5)

Theoretical Analysis

$\mathcal{X}_{\text{pool}}$: Line pool

$\mathcal{X}_{\text{plan}}$: Line plan solution

$\mathcal{X}_{\text{conflict}}$: Set of conflicting services

Proposition 1. *For any $\mathcal{X}_{conflict}$ identified in an $\mathcal{X}_{plan}$, it holds that*

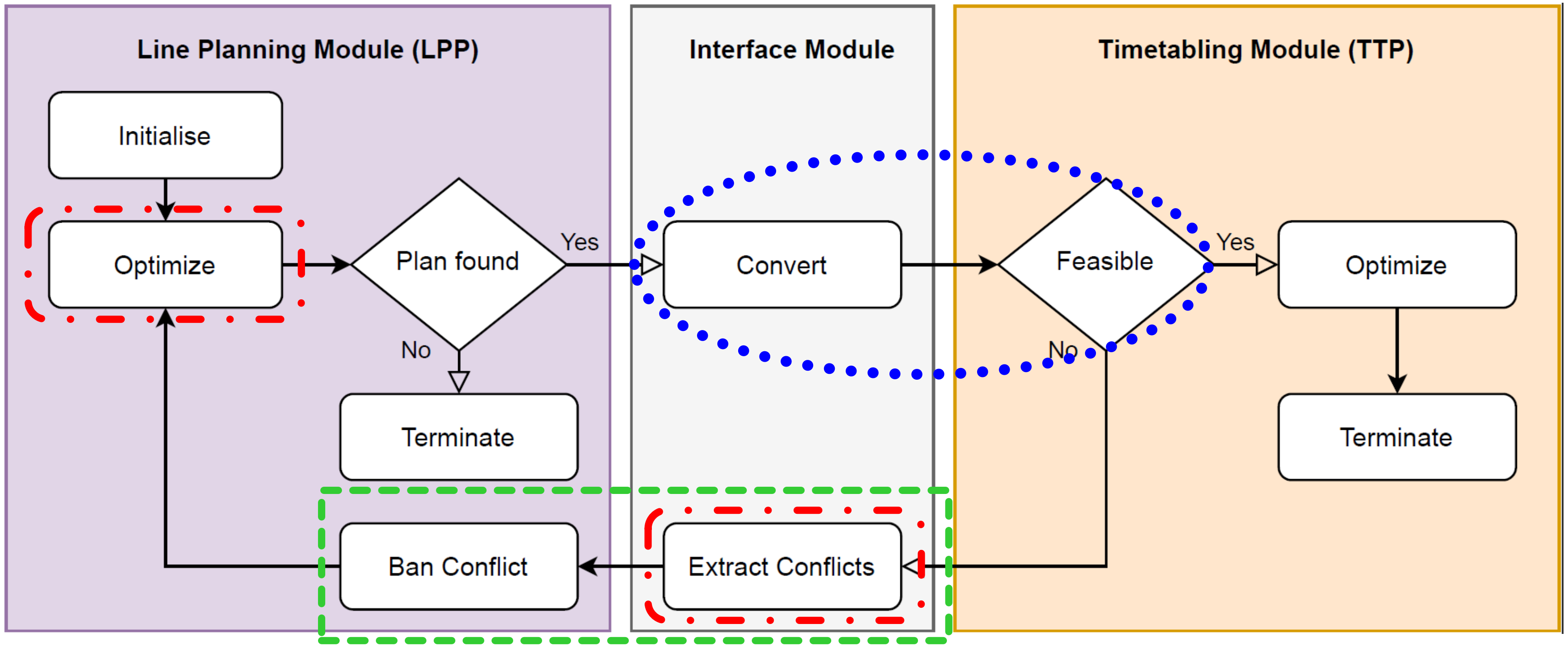
$$0 < |\mathcal{X}_{conflict}| \leq |\mathcal{X}_{plan}| \leq |\mathcal{X}_{pool}|.$$

Define $|\mathcal{X}_{\text{pool}}| = \alpha$

Proposition 2. *Banning $\mathcal{X}_{conflict}$ removes n many $\mathcal{X}_{plan}$, where $n \in [1, 2^{\alpha-1}]$.*

Proposition 3. *Instances exist such that banning $\mathcal{X}_{conflict}$ converges in $\mathcal{O}(\alpha)$ iterations whereas banning $\mathcal{X}_{plan}$ requires $\mathcal{O}(2^{\alpha})$ iterations.*

Overview of Approach



Novelty:

- Exploit infrastructure, by assigning tracks while solving
- Reduce number of iterations, by locating and banning conflicts precisely
- Reduce time per iteration, by enhancing state of the art approaches

Find a line plan such that:

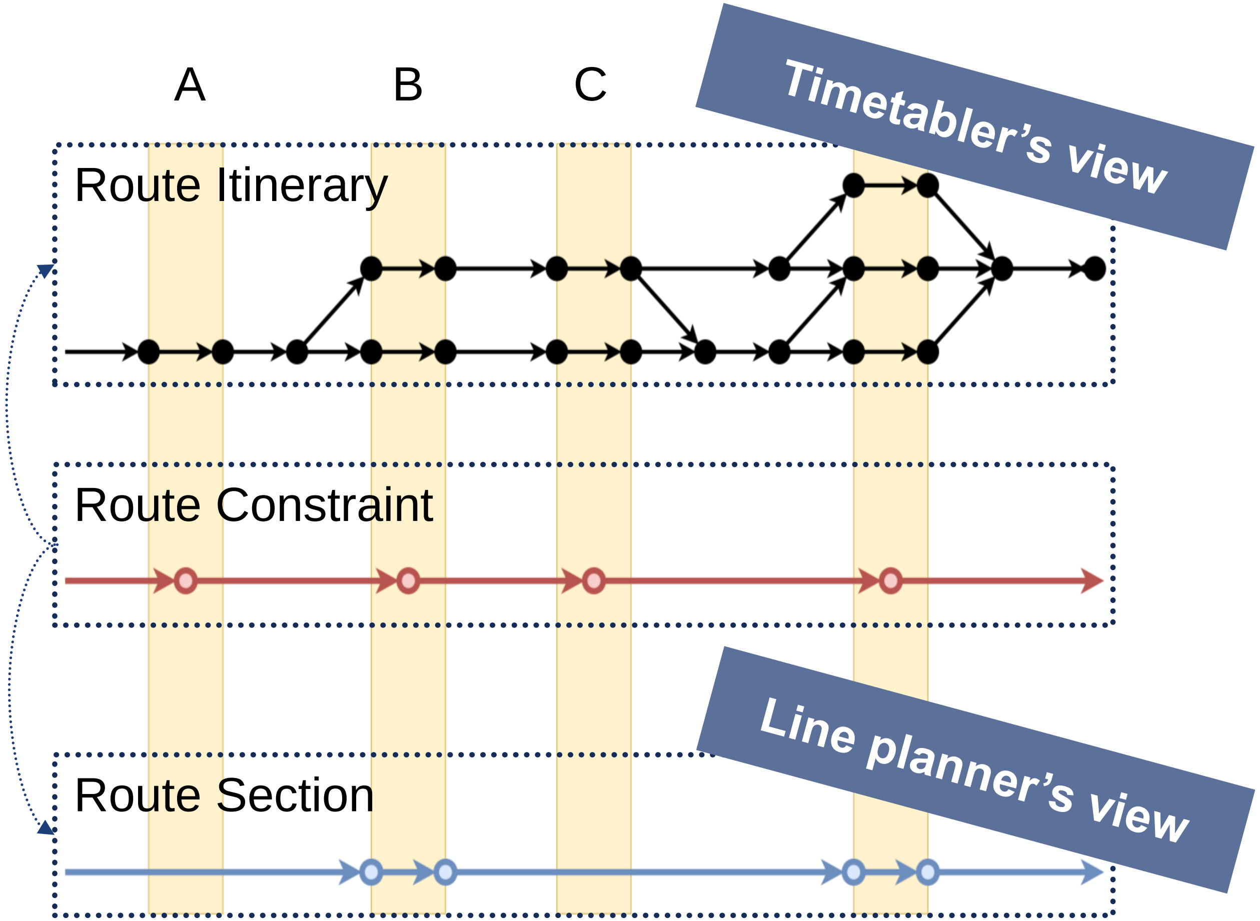
- Minimize total travel time
- Uses available vehicles
- Is free from identified conflicts

Given a line plan:

- Find a timetable
- Focus on feasibility
- Extract possible conflicts

Domain Modeling

- How to cope with different granularity?



- Route Itinerary (**RI**)
  Consists of all routing options which are available during the timetabling
- Route Constraint (**RC**)
  Defines the set of passed stations
- Route Section (**RS**)
  Required for the line planning, represents the section which is presented to the passengers

Line Planning

- Follows the MIP formulation of Burggraeve et al. (2017) and Patzold et al. (2017)

$$\min \quad \sum_{od\in\mathcal{OD}} \sum_{s\in\mathcal{S}_{od}} t_s \cdot p_s^{od} \tag{1a}$$

$$\text{subject to} \quad \sum_{x_{l,f,r}\in X_l} x_{l,f,r} \begin{cases} = 1 \text{ if } l \text{ is mandatory} \\ \leq 1 \text{ else} \end{cases} \quad \forall l \in \mathit{line} \text{ pool}, \tag{1b}$$

$$\sum_{s\in \deg^-(\mathrm{v})} p_s^{od} - \sum_{s\in \deg^+(\mathrm{v})} p_s^{od} \begin{cases} = \quad p^{od} \text{ if } v = o \\ = -p^{od} \text{ if } v = d \\ = \quad 0 \text{ else} \end{cases}, \ \forall v \in \mathcal{V}_{od}, \forall od \in \mathcal{OD}, \tag{1c}$$

$$\sum_{p^{od}\in\mathcal{OD}} p_s^{od} \leq \sum_{x_{l,f,r}\in X_s} x_{l,f,r} \cdot \text{capacity}(r) \cdot f, \ \forall s \in \mathcal{S}_l, \tag{1d}$$

$$\sum_{x_{l,f,r}\in X_r} x_{l,f,r} \cdot \text{vehicles}(f) \leq r_{\max}, \ \forall r \in \mathcal{R}, \tag{1e}$$

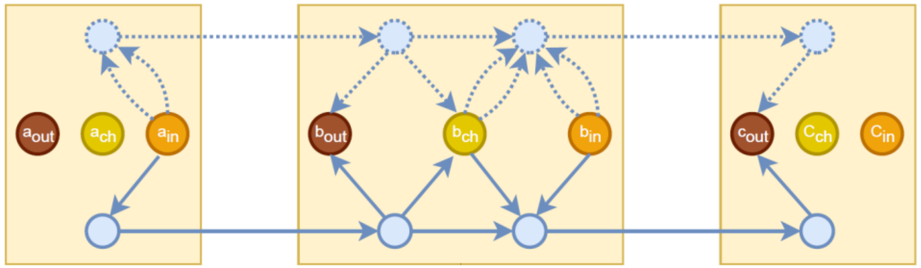
$$\sum_{x_{l,f,r}\in X_{rc}} x_{l,f,r} \cdot f \leq f_{rc,\max}, \ \forall rc \in \{RC_l\}_{l\in \mathit{line} \text{ pool}}, \tag{1f}$$

$$\sum_{x_{l,f,r}\in\mathcal{X}_{\text{conflict}}} x_{l,f,r} \leq |\mathcal{X}_{\text{conflict}}| - 1, \ \forall \mathcal{X}_{\text{conflict}} \in \mathcal{C}, \tag{1g}$$

$$x_{l,f,r} \in \{0,1\}, \forall x_{l,f,r} \in \mathcal{X}_{\text{pool}}, \tag{1h}$$

$$p_s^{od} \in \mathbb{R}^+, \ \forall s \in \mathcal{S}_{od}, \ \forall od \in \mathcal{OD}. \tag{1i}$$

- Select lines (with frequency and vehicle) minimizing travel time for all passengers
- Based on flows on a "**Section Graph**" (made of all RS plus connection nodes)



- Exclude already identified conflicts

Model Variants

LPP-prune

- LPP needs one SG per passenger OD
- Only small number of SG edges is used in a solution

↓

- Limit the maximal passengers' detour

LPP-simple

- LPP wait time is frequency-dependent

$$t_s := \begin{cases} \mathcal{T}/(2 \cdot f), & \text{if } s \in \mathcal{S}_{\text{wait}} \\ t_{s,\text{speed}} \cdot k_{\text{slack}} & \text{else} \end{cases}$$

- Higher frequencies favored, but more conflicts in TTP

↓

- Frequency-independent based on maximal frequency → Lower bound

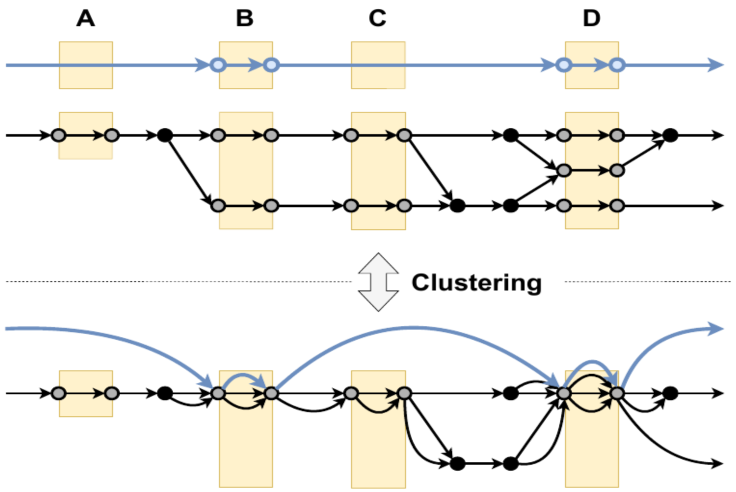
Periodic Timetabling – MIP Formulation

- Create Event Activity Network using RS-RI: $EAN = (\mathcal{E}, \mathcal{A})$
  - $e \in \mathcal{E}$ events
  - $a \in \mathcal{A}$ activities (trip, dwell, headway…)
- Assign timestamp to events so that "selectable" activities are feasible $l_a \leq t_a \leq u_a$ accounting for periodicity

$$t_a = t_{e_{a-}} - t_{e_{a+}} + k_a \mathcal{T}$$

$$
\begin{aligned}
\text{min} \quad & \sum_{a \in \mathcal{A}_S} p_a \cdot t_a & \text{(1a)}\\
\text{subject to} \quad & t_a \leq u_a + y_a(\mathcal{T} - u_a - dt), \ \forall a \in \mathcal{A}, & \text{(1b)}\\
& t_a \geq l_a - y_a \cdot l_a, \ \forall a \in \mathcal{A}, & \text{(1c)}\\
& z_{a^+} + z_{a^-} + 2 \cdot y_a \leq 2, \ \forall a \in \mathcal{A}, & \text{(1d)}\\
& \sum_{y \in \deg^+(z)} y = z, \ \forall z \in RI, \ \forall RI \in \mathcal{L}, & \text{(1e)}\\
& \sum_{y \in \deg^-(z)} y = z, \ \forall z \in RI, \ \forall RI \in \mathcal{L}, & \text{(1f)}\\
& \sum_{z \in \mathcal{Z}_{\text{source}}} z = 1, \ \forall RI \in \mathcal{L}, & \text{(1g)}\\
& t_e \in \{0, 1, .., (\mathcal{T} - dt)/dt\}, & \text{(1h)}\\
& y_a \in \{0, 1\}, \ \forall a \in \mathcal{A}, \ z \in \{0, 1\} & \text{(1i)}\\
\text{using} \quad & t_a = (t_{e_{a-}} - t_{e_{a+}}) \cdot dt + k_a \cdot \mathcal{T}, \ \forall a \in \mathcal{A}, \ k_a \in \mathbf{Z}. & \text{(1j)}
\end{aligned}
$$

- Shrink EAN by clustering arrival/ departure events



Timetabling – SAT Encoding

We encode the MIP as a Boolean satisfiability problem (SAT) because:

1. SAT is known to outperform MIP when determining feasibility (Kummling et al. 2015)
2. Locate conflicts for unfeasible instances: (SAT-solvers provide **unsatisfiable cores**)

Enhance formulation by Grossman (2011) to account for train itineraries during along the RI

$$\text{encode-event}(v) := (\neg q_{v,-dt} \wedge q_{v,\mathcal{T}-dt}) \bigwedge_{i \in \{0,dt,..,\mathcal{T}-dt\}} (\neg q_{v,i-dt} \vee q_{v,i})$$

$$\text{encode-selectable-rectangle}([i_1, i_2]\text{x}[j_1, j_2]) := \neg q_{i,i_2} \vee q_{i,i_1} \vee \neg q_{j,j_2} \vee q_{j,j_1} \vee \neg q_{z_i} \vee \neg q_{z_j}$$

$$\text{encode-}RI(z) := (\neg q_z \bigvee_{y \in \deg^+(z)} q_y) \wedge (\neg q_z \bigvee_{y \in \deg^-(z)} q_y) \wedge \text{encode-at-most-one}(\deg^+(z)) \wedge \text{encode-at-most-one}(\deg^-(z))$$

$$\text{encode-at-most-one}(\mathcal{Q}) := \bigvee_{\forall q_i, q_j \in Q,\ i<j} \neg q_i \vee \neg q_j$$

We implement a concurrent SAT approach employing different heuristics

Real Instances from RhB (CH)

Line Pool:

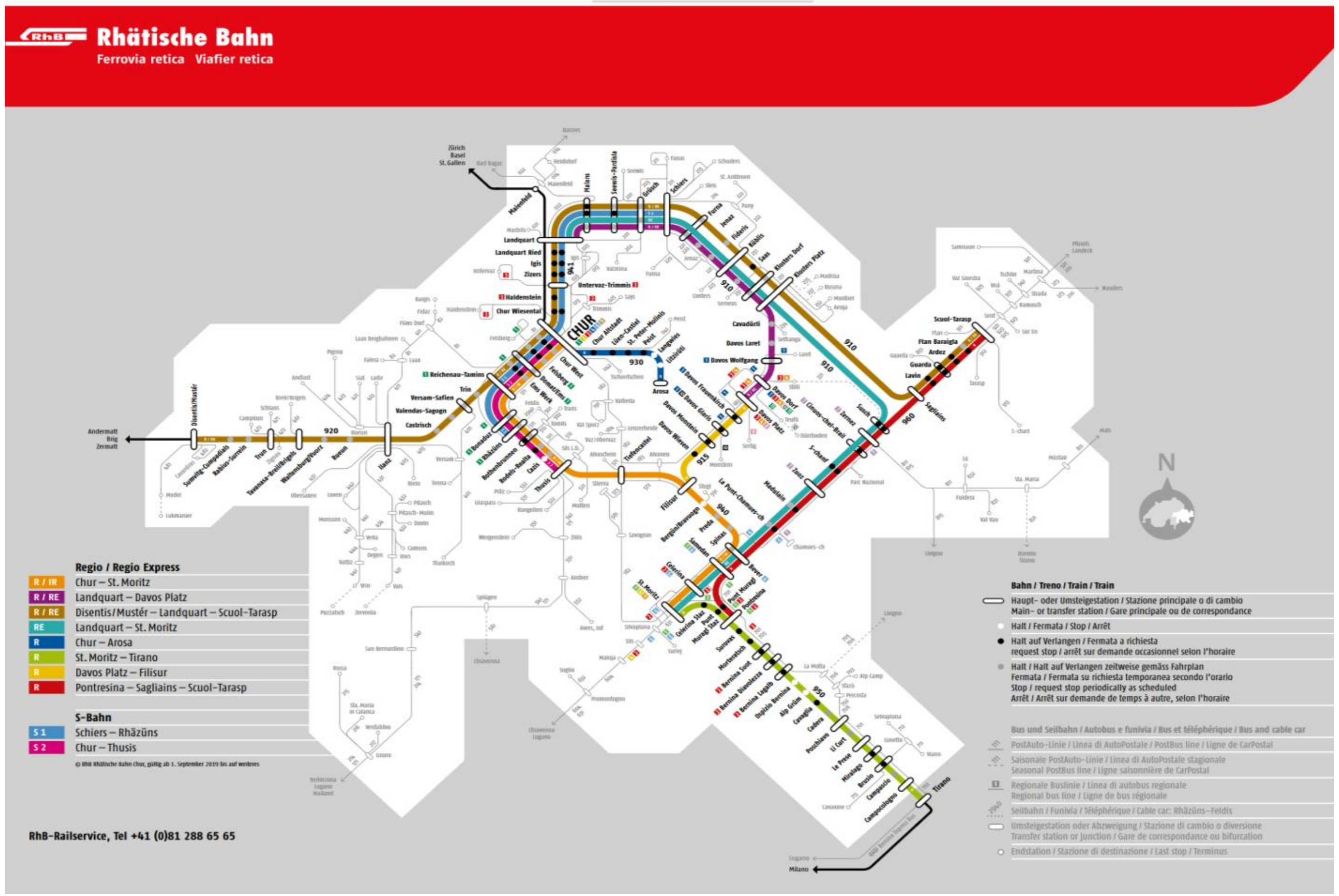
Type	Candidates	Freq. (1/h)	Mandatory
Public	42	{0, 1, 2}	✗
Freight	4	1	✓
Auto-Train	1	2	✓

Estimate trip time with constant speed

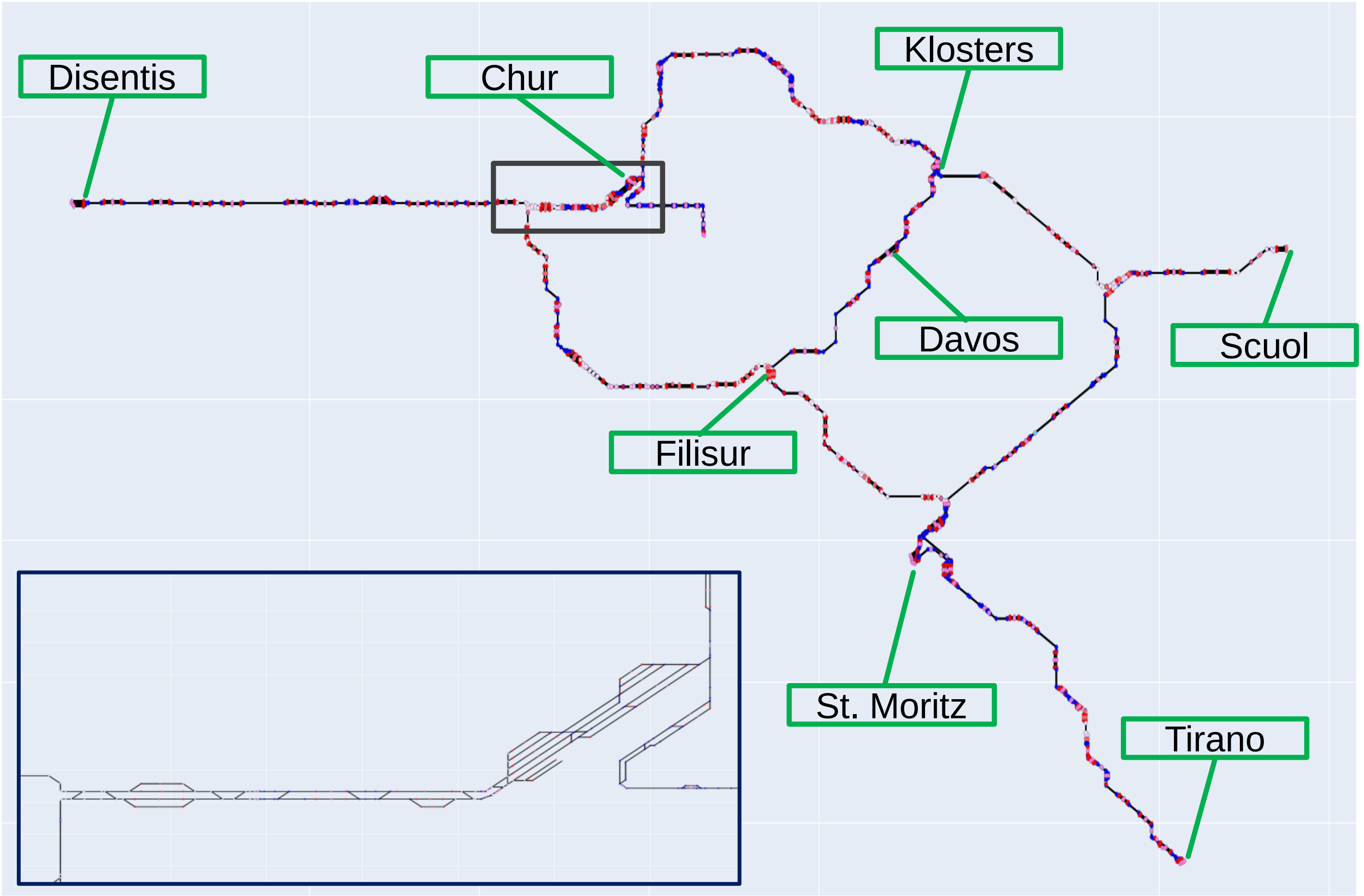
Train Class	Constant Speed
Commuter	21 m/s
Regio/Express	20 m/s
Cargo	19 m/s

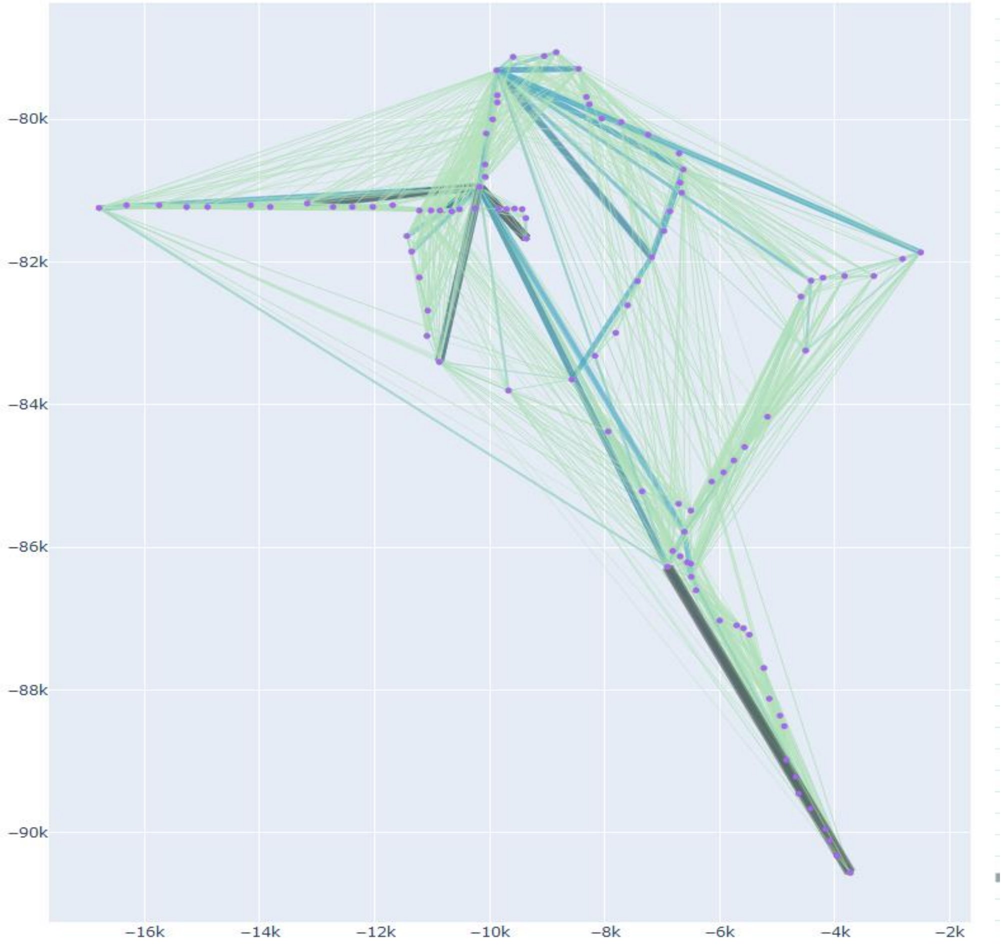
- Period: 1 hour
- Headway: 120 seconds

384 km of tracks and 102 stations



Infrastructure & Demand





Mesoscopic level, many single-track sections

1763 OD pairs

Results: Line Planning

LPP performance with three OD scenarios (Time limit: 3600 seconds, optimality gap 0.5%).

LPP model		$\mathcal{OD}_{\text{small}}$	$\mathcal{OD}_{\text{median}}$	$\mathcal{OD}_{\text{full}}$
LPP	computation time [s]	0.3	3'600.0	3'600.0
	objective value [h]	111.3	3561.2	3639.2
	optimality gap [%]	0.00	1.00	0.80
LPP-prune	computation time [s]	0.1	20.3	29.1
	objective value [h]	111.3	3554.6	3636.8
	optimality gap [%]	0.00	0.47	0.40
LPP-simple	computation time [s]	0.2	76.3	188.7
	objective value [h]	111.3	3'223.4	3299.1
	optimality gap [%]	0.00	0.25	0.41
LPP-prune-simple	computation time [s]	0.1	2.0	4.9
	objective value [h]	111.2	3232.7	3303.0
	optimality gap [%]	0.00	0.48	0.48

- Support for using **LPP-prune** (close in objective LPP and much faster)
- **LPP-prune-simple** further speeds up **LPP-prune** and can be used as a heuristic

Results: Timetabling

We test scenarios that vary the slack in activity bounds and the maximum frequencies allowed

Parameters	Approach	$\mathcal{OD}_{\text{small}}$	$\mathcal{OD}_{\text{median}}$	$\mathcal{OD}_{\text{full}}$
max slack & restrict-6-14	TTP-SAT [s]	8.6 **F**	103.0 **F**	83.5 **F**
	TTP-MIP [s]	3.1 **F**	3600.0 **U**	3600.0 **U**
	TTP-fixed-itinerary [s]	3.0 **I**	20.7 **I**	19.2 **I**
max slack & restrict-8-16	TTP-SAT [s]	8.7 **F**	17.3 **I**	15.7 **I**
	TTP-MIP [s]	3.2 **F**	153.7 **I**	23.4 **I**
	TTP-fixed-itinerary [s]	3.6 **I**	23.1 **I**	32.6 **I**
none slack & restrict-6-14	TTP-SAT [s]	9.0 **F**	14.4 **I**	26.1 **I**
	TTP-MIP [s]	5.9 **F**	6.6 **I**	3600.0 **U**
	TTP-fixed-itinerary [s]	2.9 **I**	17.4 **I**	17.2 **I**

F: feasible, **I**: infeasible, **U**: undefined (timeout)

- SAT (concurrent) significantly outperforms MIP
- Fixed-itinerary lead always to conflicts

Results: Integrated Problem

Parameters		$\mathcal{OD}_{\text{small}}$	$\mathcal{OD}_{\text{median}}$	$\mathcal{OD}_{\text{full}}$
none slack & restrict-6-14	iterations	5	17	21
	runtime [s]	111.3	1166.5	1506.8
	objective[h]	140.2	-	-
some slack & restrict-6-14	iterations	4	36	39
	runtime [s]	40.5	14’125.2	17’677.4
	objective[h]	140.2	4117.5	4217.3
max slack & restrict-8-16	iterations	4	7	7
	runtime [s]	40.8	1’170.6	3538.6
	objective[h]	140.2	3691.1	3779.3

Infeasible
(too restrictive)

Feasible, solved in
at most 5 hours

Banning conflicts w.r.t. line plans

		$\mathcal{OD}_{\text{small}}$	$\mathcal{OD}_{\text{median}}$	$\mathcal{OD}_{\text{full}}$
Ban $\mathcal{X}_{\text{plan}}$	iterations	6	179	83
max slack & restrict-8-16	runtime [s]	56	36’000	36’000
Ban $\mathcal{X}_{\text{conflict}}$	iterations	4	7	7
max slack & restrict-8-16	runtime [s]	41	1’171	3’539

- Iterations down by **12-25 times**
- Solved in **1h** vs not solved in **10h**

Conclusion

- We have studied the **joint line planning and timetabling** problem in railway, also exploiting the available **infrastructure**, i.e., accounting for train itineraries

- We have analyzed theoretically and numerically the difference between banning **line plans vs conflicts**. The latter allows to:
  - Solve more instances
  - Reduce running time (10h → 1h)
  - Reduce the number of iterations by up to 25 times

- Possible future work:
  - Lines share sections: “expand” identified conflicts to catch others affecting similar lines
  - Timetabling: Include vehicle rotations, connections, consider robustness

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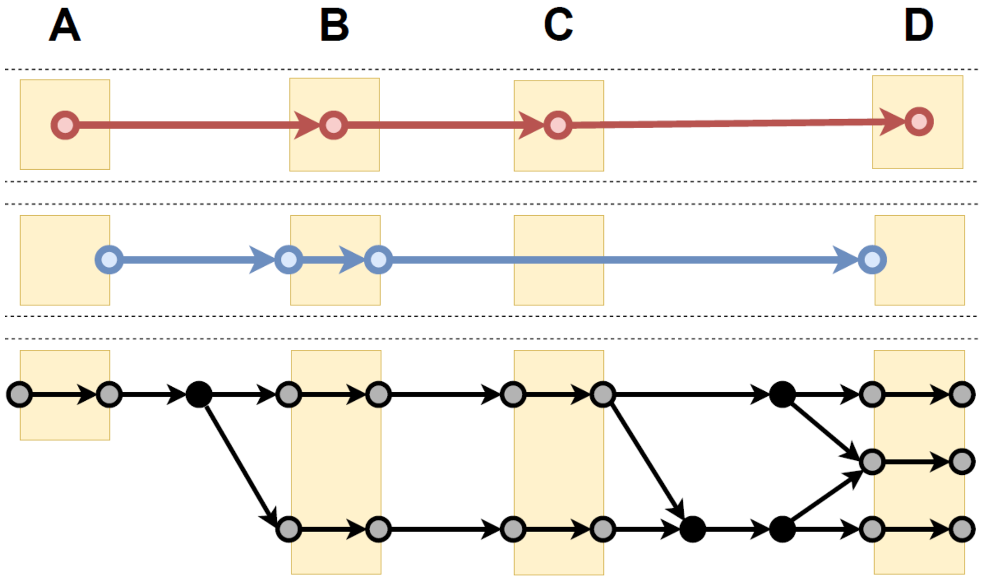

Thank you

Questions?